Coordinate Geometry

Distance Formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

e.g. Find the distance between (–1,3) and (3,5)

$$d = \sqrt{(5-3)^2 + (3+1)^2}$$

$$= \sqrt{2^2 + 4^2}$$

$$= \sqrt{20}$$

$$= 2\sqrt{5} \text{ units}$$

The distance formula is finding the length of the hypotenuse, using Pythagoras

Midpoint Formula

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

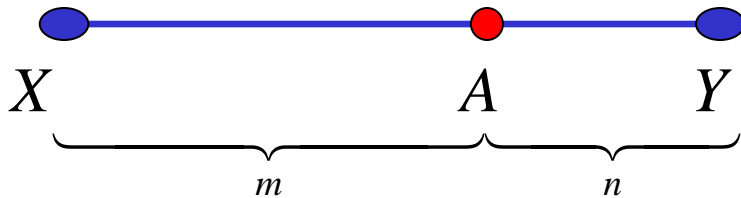e.g. Find the midpoint of (3,4) and (–2 ,1)

$$M = \left(\frac{3-2}{2}, \frac{4+1}{2} \right)$$

$$= \left(\frac{1}{2}, \frac{5}{2} \right)$$

The midpoint formula averages the x and y values

Division Of An Interval

Mathematics (2 unit) division of an interval questions are restricted to midpoint questions i.e. dividing in the ratio 1:1
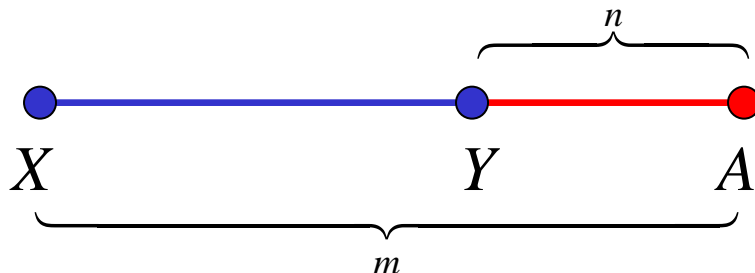
In Extension 1 you can be asked to divide an interval in a any ratio, and it could be either an internal or an external division.

A divides XY internally in the ratio $m{:}n$

OR

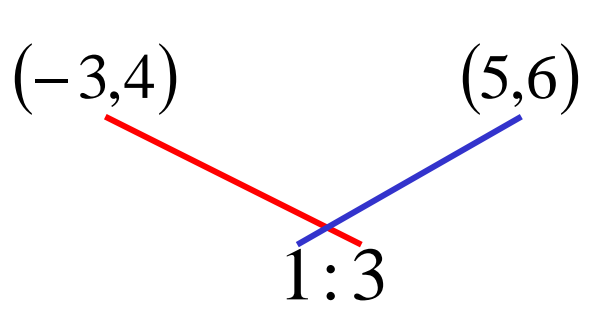A divides YX internally in the ratio $n{:}m$

A divides XY externally in the ratio $m{:}n$

Type 1: Internal Division   2003 Extension 1 HSC Q1c)

Find the coordinates of P that divides the interval joining $(-3,4)$ and $(5,6)$ internally in the ratio 1 : 3

- Write down the endpoints of the interval in the same order as they are mentioned.

- Write down the ratio.

- Draw a cross joining the ratio to the two points

- Set up your answer by drawing a set of parentheses with two vinculums separated by a comma

- Add the numbers in the ratio together to get the denominator

- Multiply along the cross and add to get the numerator

$(-3,4)$ $(5,6)$ $P = \left(\dfrac{3 \times -3 + 1 \times 5}{4}, \dfrac{3 \times 4 + 1 \times 6}{4} \right)$

$1:3$

$= \left(\dfrac{-4}{4}, \dfrac{18}{4} \right)$ $= \left(-1, \dfrac{9}{2} \right)$

Type 2: External Division   2004 Extension 1 HSC Q1c)

Let A be the point $(3,-1)$ and B be the point $(9,2)$. Find the coordinates of the point P which divides AB externally in the ratio $5:2$.

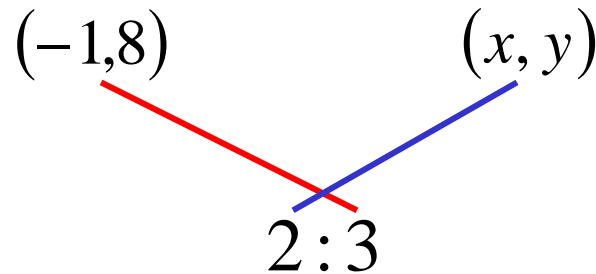- Done exactly the same as internal division, except make one of the numbers in the ratio negative

$$(3,-1) \qquad\qquad (9,2)$$

$$-5:2$$
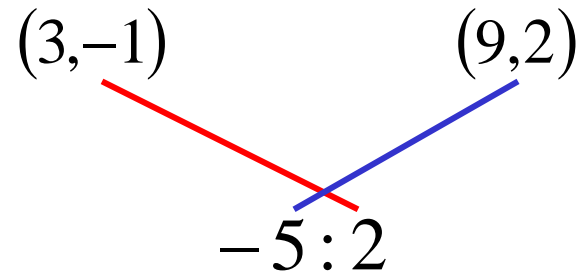
$$P = \left(\frac{2\times 3 - 5\times 9}{-3}, \frac{2\times -1 - 5\times 2}{-3} \right)$$

$$= \left(\frac{-39}{-3}, \frac{-12}{-3} \right)$$

$$= (13,4)$$

Type 3: Find an endpoint of an interval 2005 Extension 1 HSC Q1e)

The point $P(1,4)$ divides the line segment joining $A(-1,8)$ and $B(x,y)$ internally in the ratio 2 : 3.

Find the coordinates of the point B.

• Draw the endpoints, ratio and cross the same as previously

• Create the fraction for the x value and equate it with the known value

• Repeat for the y value

$$(-1,8) \qquad (x,y)$$

$$2:3$$

$$1 = \frac{3 \times -1 + 2 \times x}{5} \qquad\qquad 4 = \frac{3 \times 8 + 2 \times y}{5}$$

$$5 = -3 + 2x \qquad\qquad 20 = 24 + 2y$$
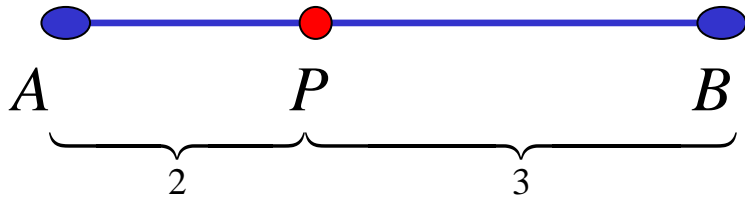
$$2x = 8 \qquad\qquad 2y = -4$$

$$x = 4 \qquad\qquad y = -2$$
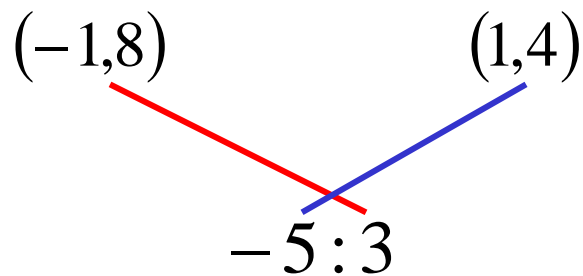
$$\therefore B = (4,-2)$$

Alternative

The point $P(1,4)$ divides the line segment joining $A(-1,8)$ and $B(x,y)$ internally in the ratio $2:3$.

Find the coordinates of the point B.

If P divides AB internally in the ratio $2:3$

Then B divides AP externally in the ratio $5:3$

$$B = \left(\frac{3 \times -1 - 5 \times 1}{-2}, \frac{3 \times 8 - 5 \times 4}{-2} \right)$$
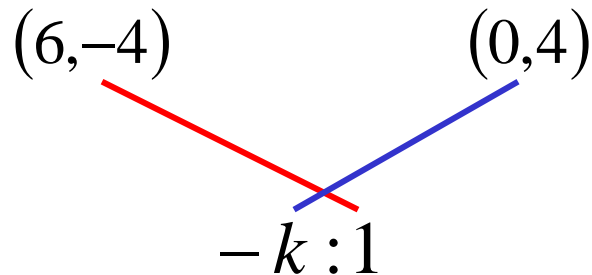
$$= \left(\frac{-8}{-2}, \frac{4}{-2} \right)$$

$$= (4, -2)$$

Type 4: Finding the ratio

The point $P(-3,8)$ divides the interval externally in the ratio $k:1$.

If A is the point $(6,-4)$ and B is the point $(0,4)$ find the value of k.

- Draw the endpoints, ratio and cross the same as usual

- Create the fraction for the either the x value or the y value (it does not matter which one) and equate it with the known value

$(6,-4)$ $(0,4)$

$-k:1$

$$-3 = \frac{1\times 6 + -k\times 0}{-k+1}$$

$$3k-3=6$$

$$3k=9$$

$$k=3$$

$\therefore P$ divides AB externally in the ratio $3:1$