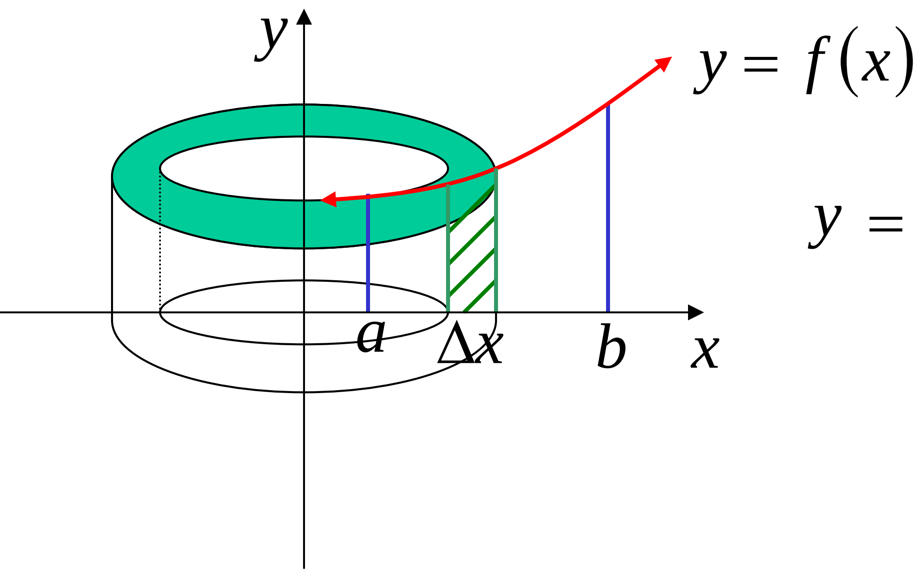
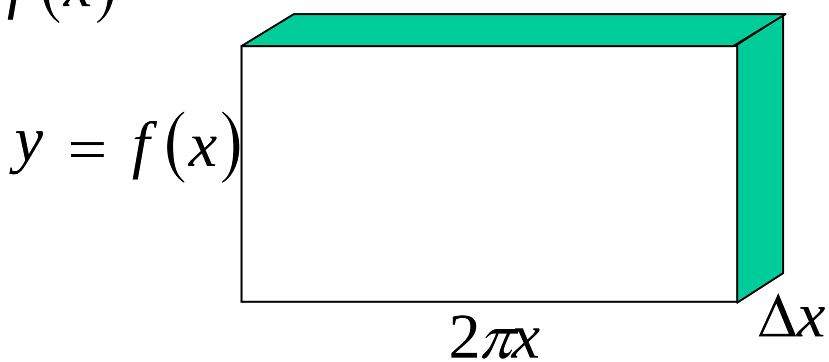
Volumes By Cylindrical Shells

$$\left(\text{slice} \parallel \text{rotation}\right)$$





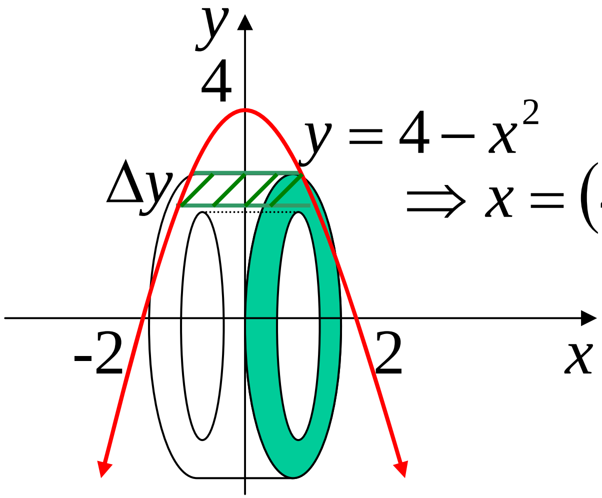
$$A(x) = 2\pi x \cdot f(x)$$

$$\Delta V = 2\pi x \cdot f(x) \cdot \Delta x$$

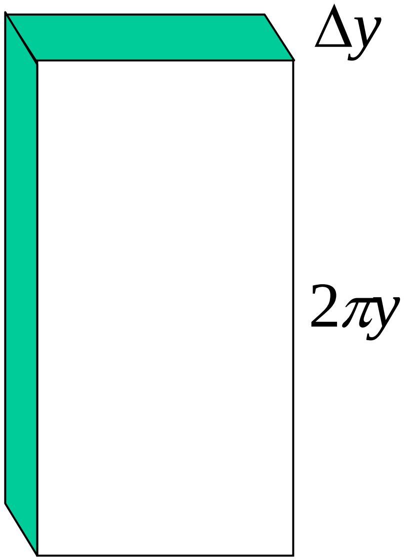
$$V = \lim_{\Delta x \to 0} \sum_{x=a}^{b} 2\pi x \cdot f(x) \cdot \Delta x$$

$$= 2\pi \int_a^b x f(x)\, dx$$

e.g. (i) Find the volume generated when the area between $y = 4 - x^2$ and the x axis is rotated about the x axis



$$y = 4 - x^2$$
$$\Rightarrow x = (4-y)^{\frac{1}{2}}$$



$$2x = 2(4-y)^{\frac{1}{2}}$$

$$A(y) = 2\pi y\left[2(4-y)^{\frac{1}{2}}\right]$$

$$\Delta V = 4\pi y(4-y)^{\frac{1}{2}} \cdot \Delta y$$

$$V = \lim_{\Delta y \to 0} \sum_{x=0}^{4} 4\pi y(4-y)^{\frac{1}{2}} \cdot \Delta y$$

$$= 4\pi \int_0^4 y(4-y)^{\frac{1}{2}}\, dy$$

$$= 4\pi \int_0^4 (4-y)y^{\frac{1}{2}}\, dy$$

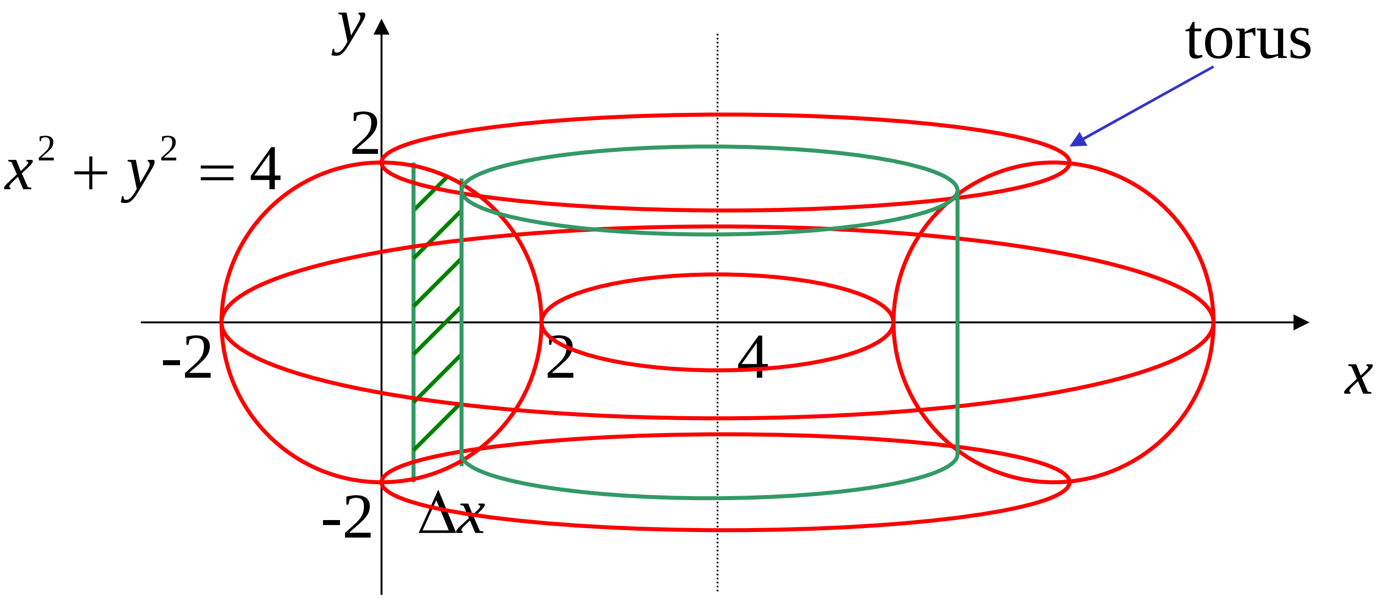
$$= 4\pi \int_0^4 \left(4y^{\frac{1}{2}} - y^{\frac{3}{2}}\right) dy$$

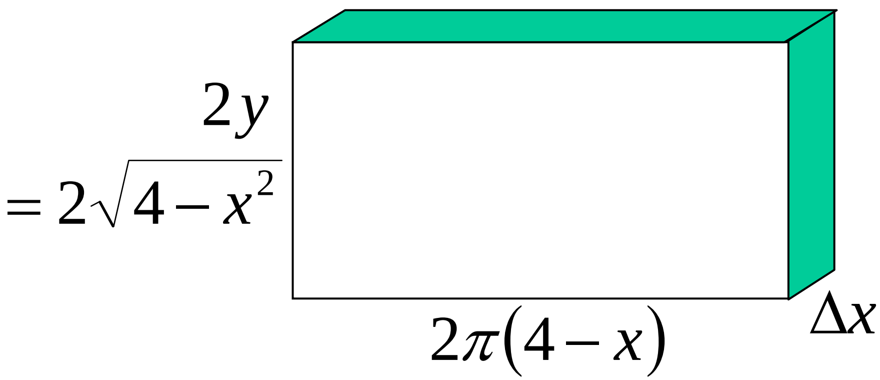
$$= 4\pi \left[\frac{8}{3}y^{\frac{3}{2}} - \frac{2}{5}y^{\frac{5}{2}}\right]_0^4$$

$$= 4\pi \left\{\frac{8}{3}(8) - \frac{2}{5}(32) - 0\right\}$$

$$= \frac{512\pi}{15} \text{ units}^3$$

(ii) Find the volume generated when $x^2 + y^2 = 4$ is rotated about the line $x = 4$





$$A(x) = 2\sqrt{4 - x^2} \cdot 2\pi(4 - x)$$

$$= 4\pi(4 - x)\sqrt{4 - x^2}$$

$$\Delta V = 4\pi(4 - x)\sqrt{4 - x^2} \cdot \Delta x$$

$$V = \lim_{\Delta x \to 0} \sum_{x=-2}^{2} 4\pi(4-x)\sqrt{4-x^2} \cdot \Delta x$$

$$= 4\pi \int_{-2}^{2} (4-x)\sqrt{4-x^2}\,dx$$

$$= 4\pi \int_{-2}^{2} 4\sqrt{4-x^2}\,dx - 4\pi \int_{-2}^{2} x\sqrt{4-x^2}\,dx$$

$$= 16\pi \int_{-2}^{2} \sqrt{4-x^2}\,dx - 4\pi \int_{-2}^{2} x\sqrt{4-x^2}\,dx$$

semi-circle $\qquad$ $\text{odd} \times \text{even} = \text{odd function}$

$$= 16\pi\left[\frac{1}{2}\pi(2)^2\right] - 0$$

$$= 32\pi^2 \text{units}^3$$

Exercise 3B;
3, 4, 6, 8

Exercise 3C;
1, 3, 9, 12, 20