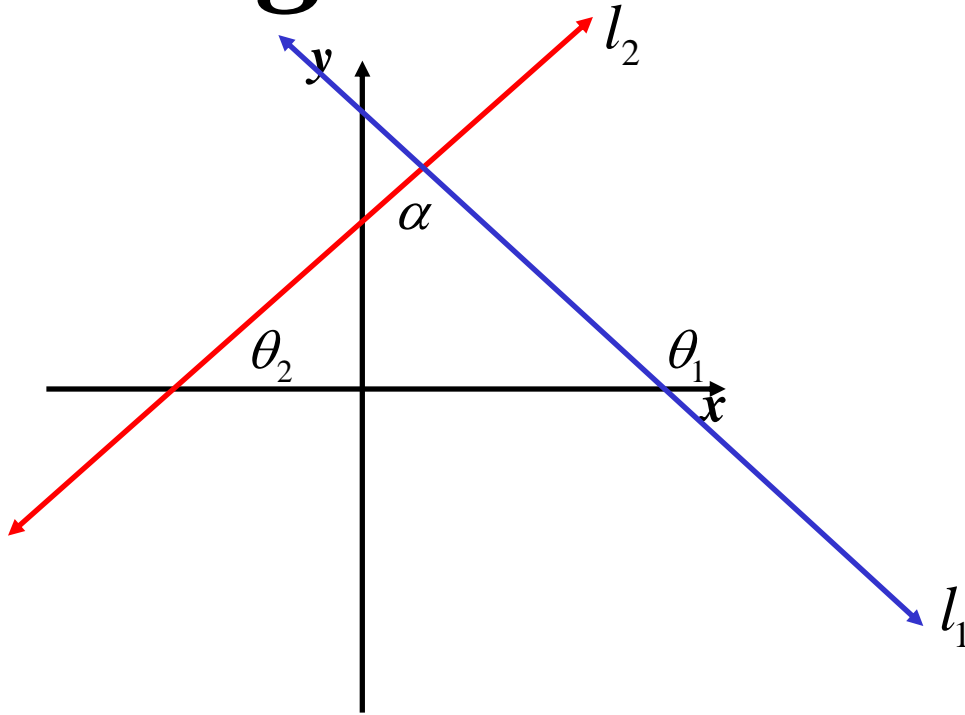


Angle Between Two Lines



l_1 has slope $m_1 = \tan \theta_1$

l_2 has slope $m_2 = \tan \theta_2$

$\theta_1 = \alpha + \theta_2$ (exterior $\angle \Delta$)

$\alpha = \theta_1 - \theta_2$

$\tan \alpha = \tan(\theta_1 - \theta_2)$

$$= \frac{\tan \theta_1 - \tan \theta_2}{1 + \tan \theta_1 \tan \theta_2}$$

$$= \frac{m_1 - m_2}{1 + m_1 m_2}$$

The acute angle between two lines with slopes m_1 and m_2 can be found using;

$$\tan \alpha = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

e.g. Find the acute angle between the lines

$$y = 2x - 1 \quad \text{and} \quad y = \frac{1}{3}x + 1$$

$$\tan \alpha = \left| \frac{2 - \frac{1}{3}}{1 + (2)\left(\frac{1}{3}\right)} \right|$$

$$= \left| \frac{6 - 1}{3 + 2} \right|$$

$$= 1$$

$$\underline{\alpha = 45^\circ}$$

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- (ii) The acute angle between the lines $y = 3x + 5$ and $y = mx + 4$ is 45° .
Find the possible values of m .

$$\tan 45^\circ = \left| \frac{3 - m}{1 + 3m} \right|$$

$$1 = \left| \frac{3 - m}{1 + 3m} \right|$$

$$|1 + 3m| = |3 - m|$$

$$1 + 3m = 3 - m \quad \text{or} \quad -(1 + 3m) = 3 - m$$

$$4m = 2$$

$$-1 - 3m = 3 - m$$

$$m = \frac{1}{2}$$

$$2m = -4$$

$$m = -2$$

$$\therefore m = \frac{1}{2} \quad \text{or} \quad m = -2$$

Exercise 14E; 1ac, 2bdf, 4, 7, 8, 10, 11a, 15*