

**EXERCISES SET 4-J**  
**FURTHER PRACTICE ON INTEGRALS**

This set should be used to consolidate the methods illustrated previously, and give practice in discriminating between them.

Find the following integrals.

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|---|---|---|
| 1. $\int \frac{x dx}{x^2+4}$                  | 2. $\int \frac{x dx}{\sqrt{x^2+4}}$       | 3. $\int \frac{5x+2}{x^2-4} dx$                               |
| 4. $\int \sin x \cos^3 x dx$                  | 5. $\int \sin x \sec^3 x dx$              | 6. $\int \cos^2 \frac{x}{2} dx$                               |
| 7. $\int x \sin x dx$                         | 8. $\int x \sec^2 2x dx$                  | 9. $\int \tan^{-1} 2x dx$                                     |
| 10. $\int \frac{x^3 dx}{x^2+1}$               | 11. $\int \frac{x dx}{(x+2)(x+4)}$        | 12. $\int \frac{(x-1)(x+1) dx}{(x-2)(x-3)}$                   |
| 13. $\int \frac{(2x-1) dx}{x^2+2x+3}$         | 14. $\int \frac{x^3 dx}{2x-1}$            | 15. $\int \frac{(1+x) dx}{\sqrt{1-x-x^2}}$                    |
| 16. $\int \frac{dx}{x^2(1-x^2)^{3/2}}$        | 17. $\int \frac{dx}{x\sqrt{a^2+x^2}}$     | 18. $\int \frac{dx}{x\sqrt{a^2-x^2}}$                         |
| 19. $\int \frac{dx}{x\sqrt{x^2-a^2}}$         | 20. $\int \frac{x dx}{\sqrt{x+1}}$        | 21. $\int \frac{\cos^{-1} x}{\sqrt{1-x^2}} dx$                |
| 22. $\int \sqrt{\frac{x+1}{x-1}} dx$          | 23. $\int \frac{dx}{x(\log x)^3}$         | 24. $\int \sec^4 3x dx$                                       |
| 25. $\int \frac{dx}{x^2(1-x)}$                | 26. $\int \frac{dx}{x^2(1+x^2)}$          | 27. $\int \frac{dx}{(1+x^2)^2}$                               |
| 28. $\int \tan^3 x dx$                        | 29. $\int \frac{dx}{5+3 \cos x}$          | 30. $\int \frac{dx}{3+5 \cos x}$                              |
| 31. $\int \frac{\sin x dx}{5+3 \cos x}$       | 32. $\int \frac{dx}{1+\cos^2 x}$          | 33. $\int \frac{dx}{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}}$ |
| 34. $\int x^2 \sin x dx$                      | 35. $\int \frac{x^2 dx}{(x-1)(x-2)(x-3)}$ | 36. $\int \frac{e^x dx}{e^x - 1}$                             |
| 37. $\int \frac{dx}{3 \sin^2 x + 5 \cos^2 x}$ | 38. $\int x^3 \cdot e^{5x^4-7} dx$        |   |
| 39. $\int x^5 \log x dx$                      | 40. $\int \frac{(3x+2) dx}{x(x+1)^3}$     | 41. $\int \log x^3 dx$  |

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| 42. $\int \frac{dx}{e^x + e^{-x}}$           | 43. $\int (5x^3+7x-1)^{3/2} \cdot (15x^2+7) dx$        |                                    |
| 44. $\int \frac{dx}{(x^2+1)(x^2+4)}$         | 45. $\int (x^2+x+1)^{-1} dx$                           | 46. $\int e^x \sin 2x dx$          |
| 47. $\int (x^2+x-1)^{-1} dx$                 | 48. $\int (x^2-x)^{-1/2} dx$                           | 49. $\int \frac{1-2x}{3+x} dx$     |
| 50. $\int x^3(4+x^2)^{-1/2} dx$              | 51. $\int \frac{\sin 2x dx}{3 \cos^2 x + 4 \sin^2 x}$  |                                    |
| 52. $\int \frac{x^2 dx}{1-x^4}$              | 53. $\int \frac{dx}{\sin x \cos x}$                    | 54. $\int \log \sqrt{x-1} dx$      |
| 55. $\int \frac{dx}{e^x-1}$                  | 56. $\int \frac{\sec^2 x dx}{\tan^2 x - 3 \tan x + 2}$ |                                    |
| 57. $\int \frac{(x+1) dx}{(x^2-3x+2)^{3/2}}$ | 58. $\int \sin 2x \cos x dx$                           | 59. $\int \frac{x dx}{1+x^2}$      |
| 60. $\int x \tan^{-1} x dx$                  | 61. $\int (1+3x+2x^2)^{-1} dx$                         | 62. $\int (9-x^2)^{3/2} dx$        |
| 63. $\int (9+x^2)^{3/2} dx$                  | 64. $\int x(9+x^2)^{3/2} dx$                           | 65. $\int \sec^2 x \tan^3 x dx$    |
| 66. $\int x^2 e^{-x} dx$                     | 67. $\int x e^{x^2} dx$                                | 68. $\int \sin x \tan x dx$        |
| 69. $\int \sin^4 x \cos^3 x dx$              | 70. $\int \frac{(x^3+1) dx}{x^3-x}$                    | 71. $\int \log(x+\sqrt{x^2-1}) dx$ |
| 72. $\int \frac{dx}{(x+1)^{1/2}(x+1)}$       |  |                                    |

Evaluate the following definite integrals, leaving results in irrational form.

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| 73. $\int_0^4 \frac{x dx}{\sqrt{x+4}}$ | 74. $\int_1^2 \frac{dx}{x(1+x^2)}$               | 75. $\int_1^2 \frac{\log x}{x} dx$  |
| 76. $\int_0^1 \cos^{-1} x dx$          | 77. $\int_1^2 \frac{(x+1) dx}{\sqrt{-2+3x-x^2}}$ | <del>78. <math>\int_0^{\pi/2} \frac{dx}{\cos^2 x + 2 \sin^2 x}</math></del> |
| 79. $\int_0^1 x\sqrt{1-x^2} dx$        | 80. $\int_2^4 x \log x dx$                       | 81. $\int_1^2 \frac{dx}{x^2+5x+4}$  |

**FURTHER PRACTICE EXERCISES**

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| 82. $\int_0^{\pi/2} (1+\sin x)^{-1} dx$      | 83. $\int_0^1 x^2 e^{-x} dx$  | 84. $\int_0^1 \frac{7+x dx}{1+x^2+x^3}$           |
| 85. $\int_0^1 \frac{e^{-2x} dx}{e^{-x} + 1}$ | 86. $\int_0^{a/2} \frac{y}{a-y} dy$   | 87. $\int_0^a \frac{(a-x)^2 dx}{a^2+x^2}$         |
| 88. $\int_0^1 \frac{(x+3) dx}{(x+2)(x+1)^2}$ | 89. $\int_0^1 \frac{x^2 dx}{x^b+1}$   | 90. $\int_0^{\pi} \cos^2 mx dx$ ,<br>m integral   |
| 91. $\int_0^{\pi/2} x \sin 2x dx$            | 92. $\int_0^{a/2} x^2 \sqrt{a^2-x^2} dx$  | 93. $\int_0^{\pi/4} \sec^2 x \tan x dx$           |
| 94. $\int_0^1 (x+2)(x^2+4x+5)^{3/2} dx$      | 95. $\int_1^2 x(\log x)^2 dx$   |   |
| 96. $\int_3^4 \frac{x^2+4}{x^2-1} dx$        | 97. $\int_1^4 \frac{x^2+4}{x(x+2)} dx$  | 98. $\int_0^{\pi/2} \frac{\cos x dx}{5-3 \sin x}$ |
| 99. $\int_0^1 \frac{dx}{(4-x^2)^{3/2}}$      | 100. $\int_0^{\pi/2} 2 \sin \theta \cos \theta (3 \sin \theta - 4 \sin^3 \theta) d\theta$ |   |