Binomial Theorem

Binomial Expansions

A binomial expression is one which contains two terms.

$$(1+x)^0 = 1$$

$$(1+x)^1 = 1+1x$$

$$(1+x)^2 = 1+2x+1x^2$$

$$(1+x)^3 = (1+x)(1+2x+1x^2)$$

$$= 1+2x+x^2+x+2x^2+x^3$$

$$= 1+3x+3x^2+x^3$$

$$(1+x)^4 = (1+x)(1+3x+3x^2+x^3)$$

$$= 1+3x+3x^2+x^3+x+3x^2+3x^3+x^4$$

$$= 1+4x+6x^2+4x^3+x^4$$

Blaise Pascal saw a pattern which we now know as **Pascal’s Triangle**

										1										
									1		1									
								1		2		1								
							1		3		3		1							
						1		4		6		4		1						
					1		5		10		10		5		1					
				1		6		15		20		15		6		1				
			1		7		21		35		35		21		7		1			
		1		8		28		56		70		56		28		8		1		
	1		9		36		84		126		126		84		36		9		1	
1		10		45		120		210		252		210		120		45		10		1

$$e.g.(i)\left(1+\frac{2x}{3}\right)^7$$

$$=1^7+7(1)^6\left(\frac{2x}{3}\right)+21(1)^5\left(\frac{2x}{3}\right)^2+35(1)^4\left(\frac{2x}{3}\right)^3+35(1)^3\left(\frac{2x}{3}\right)^4+21(1)^2\left(\frac{2x}{3}\right)^5$$

$$+7(1)\left(\frac{2x}{3}\right)^6+\left(\frac{2x}{3}\right)^7$$

$$=1+\frac{14x}{3}+\frac{84x^2}{9}+\frac{280x^3}{27}+\frac{560x^4}{81}+\frac{672x^5}{243}+\frac{448x^6}{729}+\frac{128x^7}{2187}$$

(ii) Use the expansion of $(1-x)^{10}$ to find the value of $(0.998)^{10}$ to 8 dps

$$(1-x)^{10}=1-10x+45x^2-120x^3+210x^4-252x^5+210x^6-120x^7+45x^8$$

$$-10x^9+x^{10}$$

$$(0.998)^{10}=1-10(0.002)+45(0.002)^2-120(0.002)^3$$

$$=0.98017904$$

(iii) Find the coefficient of x^2 in $(2-3x)(4+5x)^4$

$$(2-3x)(4+5x)^4$$

$$=(2-3x)\left(4^4+4(4)^3(5x)+6(4)^2(5x)^2+4(4)(5x)^3+(5x)^4\right)$$

$$\therefore \text{coefficient of } x^2 = 2(6)(4)^2(5)^2-3(4)(4)^3(5)$$

$$=4800-3840$$

$$=960$$

Exercise 5A; 2ace etc, 4, 6, 7, 9ad, 12b, 13ac, 14ace, 16a, 22, 23