Probability

Definitions

Probability: the chance of something happening

Sample Space: all possible outcomes

Equally Likely Events: events which have an equal chance of happening

Mutually Exclusive Events: only one possible outcome can occur at any one time.

e.g. a coin can be either a head or a tail, not both

Non-Mutually Exclusive Events: more than one outcome could possibly happen at any one time

e.g. a number could be both even and a multiple of three

$P(E)$: probability of E happening

$P(\overline{E})$: probability of E not happening

Probability Theory

$$0 \le P(E) \le 1$$

P(E)=0: E never happens

P(E)=1: a certain event. E must happen

$$P(E) = \frac{n(E)}{n(S)}$$

$n(E)$: the number of times E occurs

$n(S)$: total number of possibilities

e.g. A pair of dice are thrown. What is the probability that they;

(*i*) total 3?

(*ii*) total 7?

1 1	2 1	3 1	4 1	5 1	6 1
1 2	2 2	3 2	4 2	5 2	6 2
1 3	2 3	3 3	4 3	5 3	6 3
1 4	2 4	3 4	4 4	5 4	6 4
1 5	2 5	3 5	4 5	5 5	6 5
1 6	2 6	3 6	4 6	5 6	6 6

$$(i) P(=3) = \frac{2}{36}$$
$$= \frac{1}{18}$$

$$(ii) P(=7) = \frac{6}{36}$$
$$= \frac{1}{6}$$

Complementary Events

$$P(\overline{E}) = 1 - P(E)$$

e.g. What is the probability of totaling at least 3?

$$P(\geq 3) = 1 - P(1 \text{ or } 2)$$
$$= 1 - \frac{1}{36}$$
$$= \frac{35}{36}$$

Non-Mutually Exclusive Events

$$P(A \cup B) = P(A) + P(B) - P(AB)$$

e.g. What is the chance of picking an Ace or a heart from a regular deck of playing cards?

$$P(\text{Ace or heart}) = P(\text{Ace}) + P(\text{heart}) - P(\text{Ace of hearts})$$

$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$$

$$= \frac{16}{52}$$

$$= \frac{4}{13}$$

2004 Mathematics HSC Q6c)

In a game, a turn involves rolling two dice, each with faces marked 0, 1, 2, 3, 4 and 5.

The score for each turn is calculated by multiplying the two numbers uppermost on the dice.

0 X 0 = 0	1 X 0 = 0	2 X 0 = 0	3 X 0 = 0	4 X 0 = 0	5 X 0 = 0
0 X 1 = 0	1 X 1 = 1	2 X 1 = 2	3 X 1 = 3	4 X 1 = 4	5 X 1 = 5
0 X 2 = 0	1 X 2 = 2	2 X 2 = 4	3 X 2 = 6	4 X 2 = 8	5 X 2 = 10
0 X 3 = 0	1 X 3 = 3	2 X 3 = 6	3 X 3 = 9	4 X 3 = 12	5 X 3 = 15
0 X 4 = 0	1 X 4 = 4	2 X 4 = 8	3 X 4 = 12	4 X 4 = 16	5 X 4 = 20
0 X 5 = 0	1 X 5 = 5	2 X 5 = 10	3 X 5 = 15	4 X 5 = 20	5 X 5 = 25

(i) What is the probability of scoring zero on the first turn?

$$P(=0) = \frac{11}{36}$$

(ii) What is the probability of scoring 16 or more on the first turn?

$$P(\geq 16) = \frac{4}{36}$$
$$= \frac{1}{9}$$

(iii) What is the probability that the sum of the scores in the first two turns is less than 45?

$$P(<45) = 1 - P(\geq 45)$$
$$= 1 - P(20,25) - P(25,20) - P(25,25)$$
$$= 1 - \frac{2}{36} \times \frac{1}{36} - \frac{1}{36} \times \frac{2}{36} - \frac{1}{36} \times \frac{1}{36} \qquad = \frac{1291}{1296}$$

Exercise 10A; odd

Exercise 10B; odd

Exercise 10C; odd (not 23)