Greatest Coefficients and Greatest Terms

If $T_{k+1} \geq T_k$ then T_{k+1} is the greatest term

$e.g.$ For the expansion $(2+3x)^{20}$ find the greatest coefficient

$$T_{k+1} = {}^{20}C_k (2)^{20-k} (3x)^k$$

$$T_k = {}^{20}C_{k-1} (2)^{21-k} (3x)^{k-1}$$

If $T_{k+1} \geq T_k$ then T_{k+1} is the greatest term

$$ {}^{20}C_k (2)^{20-k} (3)^k \geq {}^{20}C_{k-1} (2)^{21-k} (3)^{k-1}$$

$$\frac{{}^{20}C_k (2)^{20-k} (3)^k}{{}^{20}C_{k-1} (2)^{21-k} (3)^{k-1}} \geq 1$$

$$\frac{{}^{20}C_k (2)^{20-k} (3)^k}{{}^{20}C_{k-1} (2)^{21-k} (3)^{k-1}} \geq 1$$

$$\frac{20!}{k!(20-k)!} \times \frac{(k-1)!(21-k)!}{20!} \times \frac{3}{2} \geq 1$$

$$\frac{21-k}{k} \times \frac{3}{2} \geq 1$$

$$63 - 3k \geq 2k$$

$$-5k \geq -63$$

$$k \leq \frac{63}{5}$$

$$\therefore k = 12$$

$\therefore T_{13} = {}^{20}C_{12}\, 2^8 3^{12}$ is the greatest coefficient

(ii) Find the greatest term of $(3x-4)^{15}$ when $x=\frac{1}{2}$

$$T_{k+1}={}^{15}C_k\left(\frac{3}{2}\right)^{15-k}(4)^k$$ (Ignore the negative as only concerned with magnitude)

$$T_k={}^{15}C_{k-1}\left(\frac{3}{2}\right)^{16-k}(4)^{k-1}$$

If $T_{k+1}\geq T_k$ then T_{k+1} is the greatest term

$$^{15}C_k\left(\frac{3}{2}\right)^{15-k}(4)^k\geq{}^{15}C_{k-1}\left(\frac{3}{2}\right)^{16-k}(4)^{k-1}$$

$$\frac{^{15}C_k\left(\frac{3}{2}\right)^{15-k}(4)^k}{^{15}C_{k-1}\left(\frac{3}{2}\right)^{16-k}(4)^{k-1}}\geq 1$$

$$\frac{15!}{k!(15-k)!}\times\frac{(k-1)!(16-k)!}{15!}\times\frac{4}{\frac{3}{2}}\geq 1$$

$$\frac{15!}{k!(15-k)!}\times\frac{(k-1)!(16-k)!}{15!}\times\frac{4}{\frac{3}{2}}\geq 1$$

$$\frac{16-k}{k}\times\frac{8}{3}\geq 1$$

$$128-8k\geq 3k$$

$$-11k\geq -128$$

$$k\leq\frac{128}{11}$$

$$\therefore k=11$$

$$\therefore T_{12}={}^{15}C_{11}\left(\frac{3}{2}\right)^{4}4^{11}$$

Exercise 5E; 1 to 5, 6ac, 7bd, 8b, 10

$\therefore T_{12}={}^{15}C_{11}3^{4}2^{18}$ is the greatest term