GIRRAWEEN HIGH SCHOOL
MATHEMATICS

Year 11 Extension 1 Task 2 — Monday 26th June 2006

Instructions (a) Write all answers on your own paper
(b) Show all necessary working
(c) Marks may be deducted for careless or badly arranged work
(d) Attempt all questions and start each question on a new page

Time allowed 90 minutes

Question 1 ***(17 marks)*** **Marks**

(a) Find the exact value of

(i) $\cos 15^0 \cos 30^0 - \sin 15^0 \sin 30^0$ 2.

(ii) $\sin 75^0$ 3

(iii) $\tan 15^0$ 3

(b) Simplify

(i) $\dfrac{2\tan 2\theta}{1-\tan^2 2\theta}$ 1

Expand

(ii) $\tan(\alpha+\beta)$ 2

(c) (i) Express $\cos 2\theta$ in terms of powers of $\cos\theta$. 2

(ii)Hence find the exact value of $\cos 72^0$ given that $\cos 36^0 = \dfrac{\sqrt{5}+1}{4}$ 4

Question 2 ***(16 marks)***

(a) Find the acute angle between the lines $2x - y + 5 = 0$ and $3x + 2y - 1 = 0$
To the nearest degree 3

(b) The acute angle between $2x + y - 3 = 0$ and $mx - y + 3 = 0$ is 45^0.
Find the possible value(s) of m. 4

(c) Let A (5 , 9) and B (- 2 , - 3) be two points in the number plane.
Find the point C that divides the interval

(i) internally in the ratio 2 : 1 3

(ii) externally in the ratio 3 : 2 3

(d) The point $N\left(\frac{7}{2}, \frac{-3}{2}\right)$ divides the interval M (-4 , 3) P (6, -3) internally in the ratio k : 1 . Find the value of k 3

Question 3 *(19 marks)*

(a) Given that $\sin\alpha = \frac{8}{17}$ and $\sin\beta = \frac{5}{13}$ and that $0 \le \alpha \le 90^0$ and $90^0 \le \beta \le 180^0$.

Find

(i) $\cos\alpha$ and $\cos\beta$ 4

(ii) $\sin 2\alpha$ 3

(iii) $\cos(\beta - \alpha)$ 3

(b) Solve for θ to the nearest degree, $0 \le \theta \le 360^0$

(i) $\operatorname{cosec} 2\theta = \frac{2}{\sqrt{3}}$ 2

(ii) $\sin 2\theta = \sin\theta$ 3

(iii) $3\tan\theta + 2\sec^2\theta - 7 = 0$ 4

Question 4 *(21 marks)*

(a) Prove the following identities

(i) $(\sin\theta + \cos\theta)^2 - 1 = \sin 2\theta$ 3

(ii) $\frac{\sin 2\theta}{1 + \cos 2\theta} = \tan\theta$ 3

(iii) $2\cos(45^0 + x)\cos(45^0 - x) = \cos 2x$ 3

(b) If t = $\tan\frac{\theta}{2}$ find an expression for the following in terms of t

(i) $\tan\theta + \sec\theta$ 3

(ii) $\frac{1 - \cos x}{\sin x}$ 3

(iii) $3\sin\theta + 2\cos\theta$ 3

(c) Using the t results solve for θ, $0 \le \theta \le 360^0$

$3\sin\theta + 2\cos\theta = 2$ 3

Question5 *(19 marks)*

(a) (i) Simplify $\sin(a+b) + \sin(a-b)$ 3

(ii) Hence find the exact value of $2\sin 75^0 \cos 15^0$ 3

(b)Starting with $\sin 3\theta = \sin(2\theta + \theta)$ and using the compound angle formula, show

(i) $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$ 4

(ii) Hence find the exact value of $\sin 3\theta$ if $\sin\theta = \frac{1}{\sqrt{3}}$ 4

(c) By rewriting the expression $\sqrt{3}\cos\theta - 2\sin\theta$ in the form $R\cos(\theta + \alpha)$ find values of θ, $0 \le \theta \le 360^0$ such that

$\sqrt{3}\cos\theta - 2\sin\theta = \frac{3}{2}$ 5

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Question 1.

a) i) $\cos 15 \cos 30 - \sin 15 \sin 30$

$= \cos(15+30)$

$= \cos 45$

$= \frac{1}{\sqrt{2}}$ (2)

ii) $\sin 75 = \sin(45+30)$

$= \sin 45 \cos 30 + \cos 45 \sin 30$

$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$ (3)

iii) $\tan 15 = \tan(45-30)$

$= \frac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30}$

$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \times \frac{1}{\sqrt{3}}}$

$= \frac{\sqrt{3}-1}{\sqrt{3}+1}$. (3)

(b) i) $\frac{2\tan 2\theta}{1 - \tan^2 2\theta} = \tan 4\theta$ (1)

ii) $\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta}$ (2)

(c) (i) $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \cos^2\theta - (1 - \cos^2\theta)$

$= 2\cos^2\theta - 1$. (2)

(ii) $\cos 72° = 2\cos^2 36 - 1$

$= 2\left(\frac{\sqrt{5}+1}{4}\right)^2 - 1$

$= 2\left(\frac{5+2\sqrt{5}+1}{16}\right) - 1$

$= \frac{6+2\sqrt{5}}{8} - 8$

$= 2\sqrt{5} - 2$

$= \frac{\sqrt{5}-1}{4}$ (4)

Question 2.

(a) $l_1 : 2x - y + 5 = 0$

$l_2 : 3x + 2y - 1 = 0$

$l_1 \quad y = 2x + 5 \quad m_1 = 2$

$l_2 \quad 2y = -3x + 1$

$y = -\frac{3}{2}x + \frac{1}{2} \quad m_2 = -\frac{3}{2}$

$\tan\theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$

$= \left| \frac{2 + \frac{3}{2}}{1 + 2\left(\frac{-3}{2}\right)} \right|$

$= \left| \frac{\frac{7}{2}}{-2} \right|$

$= \frac{7}{4}$

$\therefore \quad \theta = 60.25 = 60°$

(b) $m_1 = -2 \quad m_2 = m$

$\tan\theta = \left| \frac{-2 - m}{1 - 2m} \right|$

$\therefore \tan 45 = \frac{m+2}{1-2m}$ OR $\frac{m+2}{2m-1}$

$1 = \frac{m+2}{1-2m}$ OR $1 = \frac{m+2}{2m-1}$

$1 - 2m = m + 2$ OR $2m - 1 = m + 2$

$1 - 2 = 3m \qquad m = 3$

$\boxed{m = -\frac{1}{3} \text{ OR } 3}$ (4)

(c) $A(5,9) \quad B(-2,-3)$

$C_x = \frac{1 \times 5 + 2 \times (-2)}{1+2} \qquad C_y = \frac{1 \times 9 + 2 \times -3}{1+2}$

$C = \left(\frac{1}{3}, 1\right)$ (3)

(c) (ii) $C_x = \frac{-2 \times 5 + 3(-2)}{2+3}$

$C_x = -16$

$C_y = \frac{-2 \times 9 + 3(-3)}{-2+3}$

$= -27$

$C\ (-16, -27)$ (3)

(d)

$N_x = \frac{1 \times -4 + k \times 6}{1+k}$

$\frac{7}{2}(1+k) = 6k - 4$

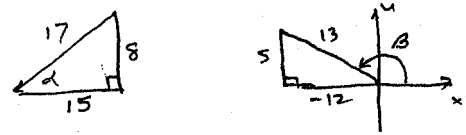
$7 + 7k = 12k - 8$

$5k = 15$

$k = 3$ (3)

Question 3

(a) $\sin\alpha = \frac{8}{17} \qquad \sin\beta = \frac{5}{13}$



(i) $\cos\alpha = \frac{15}{17} \qquad \cos\beta = -\frac{12}{13}$ (4)

(ii) $\sin 2\alpha = 2\sin\alpha\cos\alpha$

$= 2 \times \frac{8}{17} \times \frac{15}{17}$

$= \frac{240}{289}$ (3)

(iii) $\cos(\beta - \alpha) = \cos\beta\cos\alpha + \sin\beta\sin\alpha$

$= \frac{15}{17} \times \frac{-12}{13} + \frac{5}{13} \cdot \frac{8}{17}$

$= \frac{-140}{221}$ (3)

(b) (i) $\operatorname{cosec} 2\theta = \frac{2}{\sqrt{3}}$

$\sin 2\theta = \frac{\sqrt{3}}{2}$

$2\theta = 60, 120, 420, 480$

$\theta = 30, 60, 210, 240.$ (2)

(ii) $\sin 2\theta = \sin\theta$

$2\sin\theta\cos\theta = \sin\theta$

$\sin\theta(2\cos\theta - 1) = 0$

$\sin\theta = 0 \quad \theta = 0, 180, 360$

OR $\cos\theta = \frac{1}{2} \quad \theta = 60, 300$

(iii) $3\tan\theta + 2\sec^2\theta - 7 = 0$

$3\tan\theta + 2(\tan^2\theta + 1) - 7 = 0$

$2\tan^2\theta + 3\tan\theta - 5 = 0$

$(2\tan\theta + 5)(\tan\theta - 1) = 0$

$2\tan\theta = -5$

$\tan\theta = -\frac{5}{2}$

$\theta = 111.8, 291.8$

OR $\tan\theta = 1$

$\theta = 45, 225.$

$\theta = 45, 112, 225, 292$ (4)

Question 4.

(a) (i) $(\sin\theta + \cos\theta)^2 - 1 = \sin 2\theta$

L.H.S $= \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta - 1$

$= 1 + 2\sin\theta\cos\theta - 1$

$= 2\sin\theta\cos\theta$

$= \sin 2\theta$

$=$ R.H.S. (3)

Q4 cont.

(ii) $\frac{\sin 2\theta}{1+\cos 2\theta} = \tan\theta$

L.H.S. $= \frac{2\sin\theta\cos\theta}{1+\cos^2\theta-\sin^2\theta}$

$= \frac{2\sin\theta\cos\theta}{1+\cos^2\theta-(1-\cos^2\theta)}$

$= \frac{2\sin\theta\cos\theta}{2\cos^2\theta}$

$= \frac{\sin\theta}{\cos\theta}$

$= \tan\theta$

= RHS. (3)

(iii) $2\cos(45+x)\cos(45-x) = \cos 2x$

LHS $= 2\{(\cos 45\cos x - \sin 45\sin x)(\cos 45\cos x + \sin 45\sin x)\}$

$= 2\left\{\left(\frac{\cos x}{\sqrt{2}} - \frac{\sin x}{\sqrt{2}}\right)\left(\frac{\cos x}{\sqrt{2}} + \frac{\sin x}{\sqrt{2}}\right)\right\}$

$= 2\left\{\frac{\cos^2 x}{2} - \frac{\sin^2 x}{2}\right\}$

$= \cos^2 x - \sin^2 x$

$= \cos 2x$ = RHS (3)

(b) $t = \tan\theta/2$

$\therefore \tan\theta = \frac{2t}{1-t^2}$ $\sin\theta = \frac{2t}{1+t^2}$

$\cos\theta = \frac{1-t^2}{1+t^2}$

(i) $\tan\theta + \sec\theta = \tan\theta + \frac{1}{\cos\theta}$

$= \frac{2t}{1-t^2} + \frac{1+t^2}{1-t^2}$

$= \frac{1+2t+t^2}{1-t^2}$

$= \frac{(1+t)^2}{1-t^2} = \frac{1+t}{1-t}$ (3)

(ii) $\frac{1-\cos x}{\sin x} = \frac{1-\frac{1-t^2}{1+t^2}}{\frac{2t}{1+t^2}}$

$= \frac{1+t^2-(1-t^2)}{\frac{2t}{1+t^2}}$

$= \frac{2t^2}{2t}$

$= t$ (3)

(iii) $3\sin\theta + 2\cos\theta$

$= \frac{3(2t)}{1+t^2} + \frac{2(1-t^2)}{1+t^2}$

$= \frac{6t+2-2t^2}{1+t^2}$ (3)

(c) $3\sin\theta + 2\cos\theta = 2$

$\frac{2+6t-2t^2}{1+t^2} = 2$

$2+6t-2t^2 = 2+2t^2$

$0 = 4t^2 - 6t$

$0 = 2t(2t-3)$

$\therefore t = 0, 3/2$

$\tan\theta/2 = 0, 3/2$

$\theta/2 = 0, 180, 56.3°$

$\theta = 0, 112.6, 360$. (3)

Question 5.

(a) (i) $\sin(a+b) + \sin(a-b)$

$= \sin a\cos b + \cos a\sin b + (\sin a\cos b - \cos a\sin b)$

$= 2\sin a\cos b$. (3)

(ii) $2\cos 15\sin 75$

$= \sin(15+75) + \sin(75-15)$

$= \sin 90 + \sin(+60)$

$= 1 + \sqrt{3}/2$

$= \frac{2+\sqrt{3}}{2}$ (3)

(b) (i) $\sin 3\theta = \sin(2\theta+\theta)$

$= \sin 2\theta\cos\theta + \cos 2\theta\sin\theta$

$= (2\sin\theta\cos\theta)\cos\theta + (\cos^2\theta - \sin^2\theta)\sin\theta$

$= 2\sin\theta\cos^2\theta + (1-2\sin^2\theta)\sin\theta$

$= 2\sin\theta(1-\sin^2\theta) + \sin\theta - 2\sin^3\theta$

$= 2\sin\theta - 2\sin^3\theta + \sin\theta - 2\sin^3\theta$

$= 3\sin\theta - 4\sin^3\theta$. (4)

(ii) $\sin 3\theta = 3\frac{1}{\sqrt{3}} - 4\left(\frac{1}{\sqrt{3}}\right)^3$

$= \sqrt{3} - \frac{4}{3\sqrt{3}}$

$= \frac{9-4}{3\sqrt{3}}$

$= \frac{5}{3\sqrt{3}}$ (4)

(c) $\sqrt{3}\cos\theta - 2\sin\theta = R\cos(\theta+\alpha)$

$= R(\cos\theta\cos\alpha - \sin\theta\sin\alpha)$

Let $\sqrt{3}\cos\theta = R\cos\theta\cos\alpha$

$\sqrt{3} = R\cos\alpha$ (1)

$2\sin\theta = R\sin\theta\sin\alpha$

$2 = R\sin\alpha$ (2)

(2)/(1) $\tan\alpha = \frac{2}{\sqrt{3}}$

$\alpha = 49.1$

(1)^2+(2)^2 $\Rightarrow$ $R = \sqrt{7}$

$\sqrt{3}\cos\theta - 2\sin\theta = \sqrt{7}\cos(\theta+41°)$

$\sqrt{7}\cos(\theta+49) = 3/2$

$\cos(\theta+49) = \frac{3}{2\sqrt{7}}$

$\theta+49.1° = 55.46°, 304.54°$

$\theta = 6.36$ 255.44

$\theta = 6°$ $255°$ (5)