Girraween High School

Mathematics

Year 11 **June 2004**

Task 3

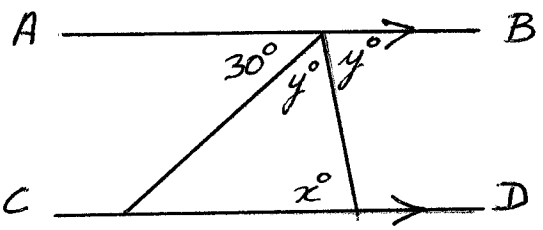
Time:90minutes

Instructions:

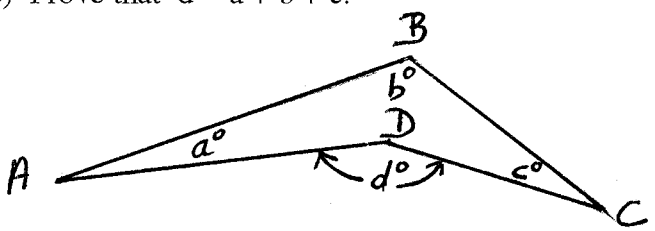
1. Attempt all questions.
2. Write your answers on your own paper.
3. All necessary working must be shown.
4. Marks will be deducted for careless or badly arranged work.

QUESTION 1 (21 marks)

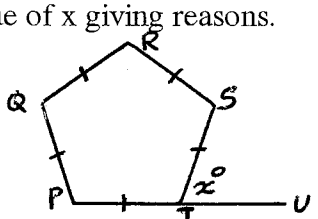
(a) Find the value of x and y. (Give reasons).



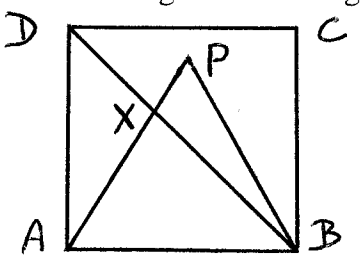
(b) Prove that d = a + b + c.



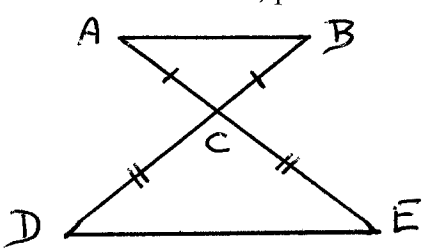
(c) Find the value of x giving reasons.



(d) In the diagram ABCD is a square and PAB is an equilateral triangle.
Calculate the size of the angle DXA in degrees, giving reasons.

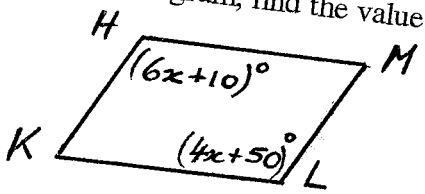


(e) Given that AC = CB and DC = CE, prove that AB is parallel to DE.

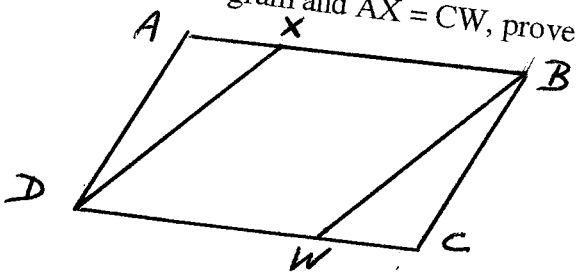


QUESTION 2 (21 marks)

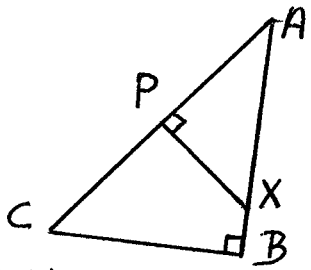
(a) If KHML is a parallelogram, find the value of x, giving reasons.



(b) If ABCD is a parallelogram and AX = CW, prove that XD = BW.



(c)



(i) Show that the triangles PAX and BAC are similar.

(ii) If AX and BC are both 5 cm, and AP is 4 cm, find PX and AC.

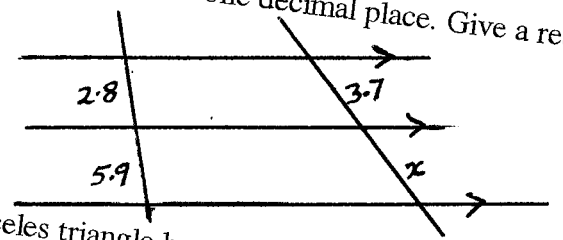
(d) The diagonals of a rhombus measure 8 cm and 15 cm. Calculate :

(i) the length of each side.

(ii) the area of the rhombus.

QUESTION 3 (15 marks).

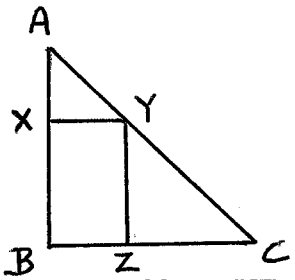
(a) Find the value of x to one decimal place. Give a reason for your answer.



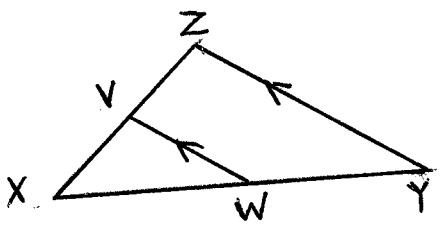
(b) An isosceles triangle has two sides 12 cm long and a perpendicular height of 10 cm.
Find the length of the base. (Leave answer in exact form).

Question 3 (continued)

(c) In the diagram XYZB is a rectangle. If AB = 8 cm, YZ = 5 cm, XY = 3 cm, find the length of BC.



(d) In the figure, XV = 12 cm, XY = 20 cm, VZ = 18 cm and WV is parallel to YZ. Find the length of XW. (Give reasons for your answer).



QUESTION 4 (**20** marks).

(a) If $f(x) = x^2 + 3x - 4$ find the values of :

(i) f(0) (ii) f(−2.5) (iii) f(x − 1)

(iv) If u(x) = f(x) − f(x − 1), show that

(α) u(x) = 2x + 2 and (β) f(2) = u(2).

(b) A function is defined by the following rule :

$$f(x) \begin{cases} = 0 & \text{if } x \le -3 \\ = -1 & \text{if } -3 < x < 0 \\ = x & \text{if } x \ge 0 \end{cases}$$

Find : (i) f(−3) + f(−2) + f(2) (ii) $f(a^2)$.

(c) Determine whether these functions are even, odd or neither. (Give reasons).

(i) $y = 1 + 2x^2$ (ii) $y = (1 + x^5)^2$

(d) Find the equation of a circle with centre (-3,2) and radius 2 units.

QUESTION 5 (17 marks).

(a) Draw a neat sketch for the following :

(i) $y = |x| + 1$ (ii) $y = -\sqrt{9 - x^2}$ (iii) $y = 2^x$.

(b) State whether or not the following are functions :

(i) $x^2 + y^2 = 9$ (ii) $y = -x^2$ (iii) $y = \frac{1}{x}$.

(c) State the domain and range for the following :

(i) $y = \frac{1}{x}$ (ii) $y = 2^x$ (iii) $y = x^2 + 2$

(d) State the domain of :

(i) $y = \sqrt{x + 1}$ (ii) $y = \sqrt{9 - x^2}$

END OF TASK.

Year 11 Mathematics Task 3, June 2004. Solutions.

1. (a) $2y + 30 = 180$ (straight $\angle$)

$\therefore 2y = 150$

$\therefore y = 75$ # (3)

$x = y$ (alternate $\angle$s, $AB \parallel CD$)

$\therefore x = 75$ # (3)

(b) $\angle ADC = 360 - d$ ($\angle$s at a point)

$\therefore a + b + c + (360 - d) = 360$ ($\angle$ sum of a quad.)

$\therefore a + b + c = d$ # (4)

(c) $5x = 360$ (sum of exterior $\angle$s)

$\therefore x = 72$ # (3)

OR

$\angle$ sum of pentagon

$= (5-2) \times 180^\circ = 540^\circ$

$\therefore$ each interior $\angle = \frac{540^\circ}{5} = 108^\circ$

$\therefore x + 108 = 180$ (straight $\angle$)

$\therefore x = 72$ #

(d) $\angle PAB = 60^\circ$ ($\angle$s of equilateral $\triangle APB$)

$\angle PAD = 30^\circ$ ($\angle DAB$ is 90° in a square)

$\angle ADB = 45^\circ$ (diagonal BD bisects $\angle ADC$)

$\therefore \angle DXA + \angle PAD + \angle ADB = 180^\circ$ ($\angle$ sum of a $\triangle$)

$\therefore \angle DXA + 30^\circ + 45^\circ = 180^\circ$

$\angle DXA + 75^\circ = 180^\circ$

$\therefore \angle DXA = 105^\circ$ # (4)

(e) In $\triangle$s ACB and ECD

$\angle ACB = \angle ECD$ (vertically opposite $\angle$s)

$\frac{AC}{CE} = \frac{BC}{DC}$ (same ratio)

$\therefore \triangle ACB \parallel\parallel \triangle ECD$ (two sides of one $\triangle$ are proportional to 2 sides of another $\triangle$ and included $\angle$s are equal) (4)

$\therefore \angle ABC = \angle EDC$ (corresponding $\angle$s of similar $\triangle$s)

But these are alternate angles, $\therefore AB \parallel DE$ #.

2. (a) $6x + 10 = 4x + 50$ (opposite $\angle$s of $\parallel$ gram)

$\therefore 6x - 4x = 50 - 10$

$2x = 40$ (3)

$\therefore x = 20$ #

(b) In $\triangle$s AXD and CWB

$AX = CW$ (given)

$AD = CB$ (opposite sides of $\parallel$ gram)

$\angle XAD = \angle WCB$ (opposite $\angle$s of $\parallel$ gram)

$\therefore \triangle AXD \equiv \triangle CWB$ (SAS)

$\therefore XD = WB$ (corresponding sides of congruent $\triangle$s)

(5) #

2 (c) (i) In $\triangle$s PAX and BAC

$\angle A$ is common

$\angle P = \angle B$ (right $\angle$) (3)

$\therefore \triangle PAX \,|||\, \triangle BAC$ (equiangular)

(ii) $PX^2 = 5^2 - 4^2$

$= 25 - 16$

$= 9$ (2)

$\therefore PX = 3$ cm #

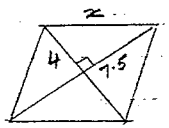
$\frac{AX}{AC} = \frac{PX}{BC}$ (same ratio)

$\therefore \frac{5}{AC} = \frac{3}{5}$ (3)

$\therefore 3AC = 25$

$\therefore AC = \frac{25}{3}$ or $8\frac{1}{3}$ cm #

(d)



(i) $x^2 = 4^2 + 7.5^2$ (3)

$= 16 + 56.25$

$= 72.25$

$\therefore x = \sqrt{72.25} = 8.5$ cm #

(ii) Area $= \frac{1}{2} \times 8 \times 15 = 60\ cm^2$ # (2)

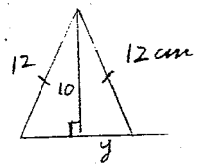
3. (a) $\frac{2.8}{5.9} = \frac{3.7}{x}$ (ratios of intercepts)

$\therefore 2.8x = 3.7 \times 5.9$

$x = \frac{3.7 \times 5.9}{2.8}$ (3)

$x = 7.8$ (1 dec. pl.) #

(b)



(3)

$\therefore y^2 = 12^2 - 10^2 = 144 - 100$

$y = \sqrt{44} = 2\sqrt{11}$

$\therefore$ base $= 2y$

$= 4\sqrt{11}$ cm. #

(c) In $\triangle$s AXY and ABC

$\angle A$ is common

$\angle AXY = \angle ABC = 90^\circ$

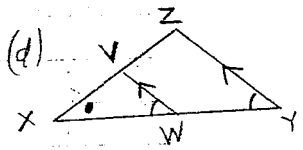
$\therefore \triangle AXY \,|||\, \triangle ABC$ (equiangular)

$\therefore \frac{AX}{AB} = \frac{XY}{BC}$ (corresponding sides in same ratio)

$\therefore \frac{3}{8} = \frac{3}{BC}$ (4)

$\therefore BC = 8$ cm #

(d)



In $\triangle$s XWV and XYZ (5)

$\angle X$ is common

$\angle W = \angle Y$ (corresponding $\angle$s, $VW \parallel ZY$)

$\therefore \triangle XWV \,|||\, \triangle XYZ$

$\therefore \frac{XW}{XY} = \frac{XV}{XZ}$ (corresponding sides in same ratio)

$\therefore \frac{XW}{20} = \frac{12}{30}\ ^2_5$

$\therefore 5XW = 40, \quad \therefore XW = 8$ cm #

4. (a) $f(x) = x^2 + 3x - 4$

(i) $f(0) = -4$ # (1)

(ii) $f(-\frac{5}{2}) = (-\frac{5}{2})^2 + 3(-\frac{5}{2}) - 4$

$= \frac{25}{4} - \frac{15}{2} - 4$

$= -5\frac{1}{4}$ # (2)

(iii) $f(x-1) = (x-1)^2 + 3(x-1) - 4$

$= x^2 - 2x + 1 + 3x - 3 - 4$

$= x^2 + x - 6$. # (2)

(iv) $u(x) = f(x) - f(x-1)$

(α) $u(x) = x^2 + 3x - 4 - (x^2 + x - 6)$

$= x^2 + 3x - 4 - x^2 - x + 6$

$= 2x + 2$ # (2)

(β) $f(2) = 2^2 + 3(2) - 4 = 6$

$u(2) = 2(2) + 2 = 6$ (2)

$\therefore f(2) = u(2)$ #

(b) (i) $f(-3) + f(-2) + f(2)$

$= 0 + (-1) + 2$

$= 1$ # (2)

(ii) $f(a^2) = a^2$ # (1)

(c) (i) $f(x) = 1 + 2x^2$

$f(-x) = 1 + 2(-x)^2$

$= 1 + 2x^2$ (3)

$f(x) = f(-x)$ $\therefore$ even function #

(ii) $f(x) = (1 + x^5)^2$

$f(-x) = (1 - x^5)^2$

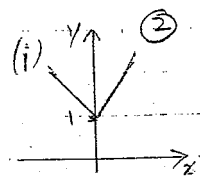
$-f(x) = -(1 + x^5)^2$ (3)

$\therefore$ neither odd nor even function. #

(d) $(x+3)^2 + (y-2)^2 = 4$ # (2)

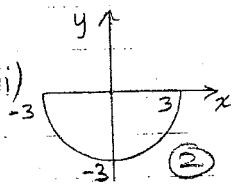
5. (a)

(i)



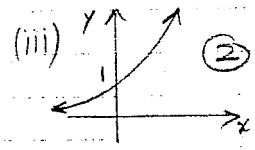
(2)

(ii)



(2)

(iii)



(2)

(b) (i) NO (ii) YES (iii) YES (3)

(c) (i) $D: \{x : \text{all } x, x \neq 0\}$ (1)

$R: \{y : \text{all } y, y \neq 0\}$ (1)

(ii) D: all real x (1)

R: $y > 0$ (1)

(iii) $D: \{x : \text{all real } x\}$ (1)

$R: \{y : y \geq 2\}$ (1)

(d) (i) $D: \{x : x \geq -1\}$ (1)

(ii) $D: \{x : -3 \leq x \leq 3\}$ (1)