GIRRAWEEN HIGH SCHOOL

YEAR 11

Mathematics

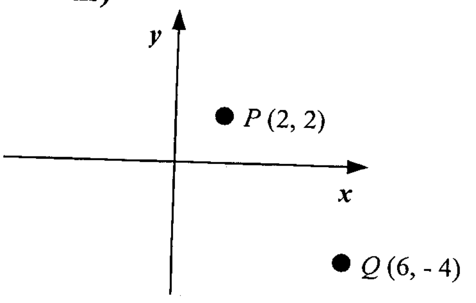
Task 4

August, 2004

TIME: 90mins

INSTRUCTIONS:
1. Attempt all questions.
2. Write your answers on your own paper.
3. All necessary working must be shown.
4. Marks will be deducted for careless or badly arranged work.

Question 1 **(16 marks)**



(a) If P is the point (2, 2) and Q is the point (6, - 4).
Find,
(i) The gradient of the line PQ. 2

(ii) The equation of the line PQ 3

(iii) The midpoint of PQ. 2

(iv) The exact distance from P to Q. 3

(b) What is the perpendicular distance of the point (-1, -2) from the line $6x + 5y = 30$. 3

(c) Find the equation of the line that has an inclination of 135° with the x axis and a y intercept of -2. 3

Question 2 **(14 marks)**

(a) find the equation of the line perpendicular to $2x + y = 6$ that passes through (1, 3). 4

(b) Find the equation of the line that passes through the point of intersection of the lines $2x - 3y + 5 = 0$ and $6x - y + 3 = 0$ and also through the point (-2, -1). 5

(c) The equations of the sides of a triangle are $2x - 3y + 5 = 0$, $2x + y = 7$ and $2x + 5y = 3$. Find the coordinates of its vertices. 5

Question 3 **(12 marks)**

Evaluate,

(a) $\lim\limits_{x \to 1} \dfrac{x-4}{2x-8}$ (b) $\lim\limits_{x \to 2} \dfrac{x^2-4}{x^2+2x-8}$ 4

(c) $\lim\limits_{x \to \infty} \left(\dfrac{1}{x+2} \right)$ (d) $\lim\limits_{x \to \infty} \dfrac{3x^2+x-3}{2x^2-x-2}$ 4

(e) (i) If $f(x) = x^2 + x$, evaluate $f(x+h) - f(x)$ 2

(ii) Differentiate $y = x^2 + x$ from first principles. 2

Question 4 **(15 marks)**

Differentiate the following

(a) $y = x^7 + 3x^2 - 4$ (b) $y = (3x+7)^4$ 4

(c) $y = \dfrac{x-1}{x-2}$ (d) $y = \sqrt[3]{x}$ 5

(e) $y = 3x^4 + \dfrac{2}{x}$ (f) $y = \dfrac{2x^4 - 3x^3 + x}{x}$ 6

Question 5 **(23 marks)**

Differentiate the following

(a) $y = (3x^2 - 2)(5+x)^3$ (b) $y = \dfrac{2}{(x^2+3)^4}$ 6

(c) $y = \dfrac{2-x^3}{7x^2+x}$ (d) $y = 6x\sqrt{4-x^2}$ 7

(e) Find the equation of the tangent and normal at $x = 3$ for the curve $y = x(2-x)$. 5

(f) Find the point on the curve $y = \sqrt{x-2}$ where the tangent is parallel to the line $2x - 2y + 3 = 0$. 5

*Mathematics – TASK 4
(2004) Yr 11.

Q1 (a) $P(2,2)$ $Q(6,-4)$

(i) $m = \frac{-4-2}{6-2} = \frac{-6}{4} = -\frac{3}{2}$ (2)

(ii) $y-2 = -\frac{3}{2}(x-2)$ or $2y-4=-3x+6$ (3)

$y = -\frac{3}{2}x+5$ $\quad 3x+2y-10=0$

(iii) Midpoint $(4,-1)$ (2)

(iv) $d = \sqrt{(6-2)^2+(-4-2)^2}$

$= \sqrt{16+36}$

$d = \sqrt{52}$ or $= 2\sqrt{13}$ (3)

(b) $d = \frac{|ax+by+c|}{\sqrt{a^2+b^2}}$

$= \frac{|6(-1)+5(-2)+(-30)|}{\sqrt{6^2+5^2}}$

$= \frac{|-46|}{\sqrt{61}} = \frac{46}{\sqrt{61}}$ (3)

(c) $m = \tan 135$

$m = -1$ $\quad b = -2$ (3)

$\therefore y = -x-2$ or $x+y+2=0$

Q2 (a) $m_1 = -2$ $\quad \therefore m_2 = \frac{1}{2}$ $\quad (1,3)$

$\therefore y-3 = \frac{1}{2}(x-1)$

$y = \frac{1}{2}x+2\frac{1}{2}$

or $x-2y+5=0$ (4)

Q2 (b) $2x-3y+5=0$ ① $(\times 3)$

$6x-y+3=0$ ②

$6x-9y+15=0$ ③

③ − ②

$-8y+12=0$

$y = \frac{12}{8}$

$y = 1\frac{1}{2}$

$\therefore 2x-3(1\frac{1}{2})+5=0$

$2x+\frac{1}{2}=0$

$2x = -\frac{1}{2}$

$x = -\frac{1}{4}$ $\quad (-\frac{1}{4}, 1\frac{1}{2})$

& $(-2,-1)$

$\frac{y+1}{x+2} = \frac{1\frac{1}{2}+1}{-\frac{1}{4}+2}$

$\frac{y+1}{x+2} = \frac{\frac{5}{2}}{\frac{7}{4}}$

$y+1 = \frac{10}{7}(x+2)$

$7y+7 = 10x+20$ $\quad$ or $y = \frac{10}{7}x+\frac{13}{7}$

$10x-7y+13=0$ (5)

OR (b) $(-2,-1)$ $\quad$ K-METHOD

$2x-3y+5+k(6x-y+3)=0$

$2(-2)-3(-1)+5+k(6(-2)-(-1)+3)=0$

$4+k(-8)=0$

$k = \frac{1}{2}$

$\therefore 2x-3y+5+\frac{1}{2}(6x-y+3)=0$

$4x-6y+10+6x-y+3=0$

$10x-7y+13=0$

Q2 (b) $2x-3y+5=0$ ①

$2x+y-7=0$ ②

$2x+5y-3=0$ ③

① − ②

$-4y+12=0$ $\quad$ y = 3 into ①

$y=3$ $\quad 2x-3(3)+5=0$

$2x-4=0$

$x=2$

$(2,3)$

③ − ②

$4y+4=0$ $\quad$ y = −1 into ②

$y=-1$ $\quad 2x+(-1)-7=0$

$2x-8=0$

$x=4$

$(4,-1)$

③ − ①

$8y-8=0$ $\quad$ y = 1 into ③

$y=1$ $\quad 2x+5(1)-3=0$

$2x+2=0$

$x=-1$

$(-1,1)$

(5)

Q3

a) $\frac{1-4}{2(1)-8} = \frac{-3}{-6} = \frac{1}{2}$ (2)

b) $\lim_{x \to 2} \frac{(x+2)\cancel{(x-2)}}{(x+4)\cancel{(x-2)}}$

$= \frac{2+2}{2+4} = \frac{4}{6} = \frac{2}{3}$ (2)

c) $\lim_{x \to \infty} \frac{\frac{1}{x}}{1+\frac{2}{x}} = \frac{0}{1} = 0$ (2)

d) $\lim_{x \to \infty} \frac{3 + \frac{1}{\infty} + \frac{3}{\infty^2}}{2 - \frac{1}{\infty} - \frac{2}{\infty^2}} = \frac{3}{2}$ (2)

e) (i) $f(x+h) = (x+h)^2 + (x+h)$

$= x^2 + 2xh + h^2 + x + h$

$f(x) = x^2 + x$

$\therefore f(x+h) - f(x) = 2xh + h^2 + h$

$= h(2x+h+1)$ (2)

(ii) $\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{\cancel{h}(2x+h+1)}{\cancel{h}}$

$= 2x + 0 + 1$

$\frac{dy}{dx} = 2x+1$ (2)

Q4

a) $y' = 7x^6 + 6$ (2)

b) $y' = 12(3x+7)^3$ (2)

c) $y' = \frac{(x-2).1 - (x-1).1}{(x-2)^2}$

$= \frac{-1}{(x-2)^2}$ (3)

d) $y' = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3\sqrt[3]{x^2}}$ (2)

e) $y' = 12x^3 - 2x^{-2}$

$= 12x^3 - \frac{2}{x^2}$ (3)

f) $y = 2x^3 - 3x^2 + 1$

$y' = 6x^2 - 6x$ (3)

Q5

a) $y' = (3x^2-2).3(5+x)^2 + (5+x)^3.6x$

$y' = 3(5+x)^2[(3x^2-2) + 2x(5+x)]$

$y' = 3(5+x)^2(5x^2+10x-2)$ (3)

b) $y = 2(x^2+3)^{-4}$

$y' = -8(x^2+3)^{-5}.2x$

$= \frac{-16x}{(x^2+3)^5}$ (3)

c) $y' = \frac{(7x^2+x).-3x^2 - (2-x^3)(14x+1)}{(7x^2+x)^2}$

$y' = \frac{-21x^4 - 3x^3 - (28x + 2 - 14x^4 - x^3)}{(7x^2+x)^2}$

$y' = \frac{-7x^4 - 2x^3 - 28x - 2}{(7x^2+x)^2}$ (3)

Q5

d) $y' = uv' + vu'$

$y' = 6x.\frac{1}{2}(4-x^2)^{-\frac{1}{2}}x(-2x)$

$+ (4-x^2)^{\frac{1}{2}}.6$

$= -6x^2(4-x^2)^{-\frac{1}{2}} + 6(4-x^2)^{\frac{1}{2}}$

$= 6(4-x^2)^{-\frac{1}{2}}[-x^2 + (4-x^2)]$

$= \frac{6}{\sqrt{4-x^2}}.(4-2x^2)$

$= \frac{24-12x^2}{\sqrt{4-x^2}}$ (4)

(e) $y = x(2-x)$

$y = -x^2 + 2x$

$\frac{dy}{dx} = -2x + 2$

when $x=3$

$y = 3(2-3)$

$y = -3$

when $x=3$ Tangent

$\frac{dy}{dx} = -2(3) + 2$

$m_1 = -4$

$\therefore$ Normal

$m_2 = \frac{1}{4}$.

$\therefore (3,-3), m = -4$

$y + 3 = -4(x-3)$

$y = -4x + 9$

$(3,-3), m = \frac{1}{4}$

$y + 3 = \frac{1}{4}(x-3)$

$y = \frac{1}{4}x + 2\frac{1}{4}$

(5)

(f) $y = (x-2)^{\frac{1}{2}}$

$2x - 2y + 3 = 0$

$y = x + \frac{3}{2}$

$\therefore m = 1$

$\frac{dy}{dx} = \frac{1}{2}(x-2)^{-\frac{1}{2}}.1$

$\frac{dy}{dx} = \frac{1}{2\sqrt{x-2}}$

$\therefore$ when $\frac{dy}{dx} = 1$

$1 = \frac{1}{2\sqrt{x-2}}$

$\sqrt{x-2} = \frac{1}{2}$

$x - 2 = \frac{1}{4}$

$x = 2\frac{1}{4}$

$\therefore y = (2\frac{1}{4} - 2)^{\frac{1}{2}}$

$y = \sqrt{\frac{1}{4}}$

$y = \frac{1}{2}$

$\therefore (2\frac{1}{4}, \frac{1}{2})$ (5)