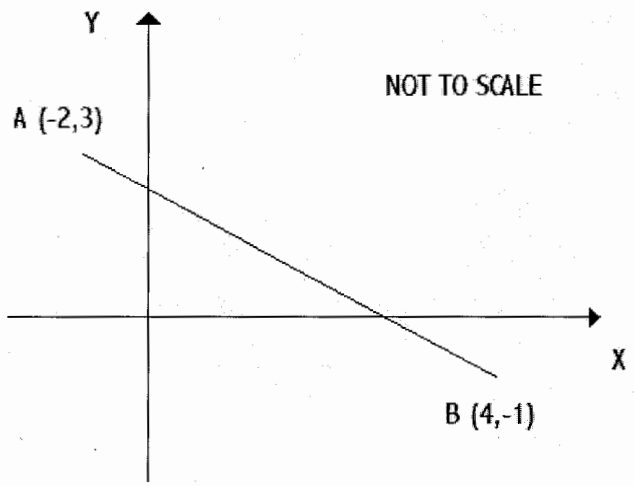
GIRRAWEEN HIGH SCHOOL

MATHEMATICS

Year 11 **14th August 2006**
Task 4 **Time allowed 90 minutes**

INSTRUCTIONS : Attempt all 6 questions..
Write your answers on your own paper.
All necessary working must be shown.
Marks may be deducted for careless or poorly arranged work.
Begin each question on a new page.

Question 1. (11 Marks) **Marks**



In the diagram above, the interval AB joining the points A (-2, 3) and B (4.-1) is shown. Find

(a) The gradient of the line AB. **2**
(b) The equation of the line AB **2**
(c) The distance AB (leave your answer as a surd) **2**
(d) The mid point M of AB **1**
(e) The equation of the line through M perpendicular to AB **2**
(f) Find the angle that the line CM makes with the x axis (nearest degree) **2**

Qustion 2 (14 Marks)

(a) Find the perpendicular distance from the point (-1 , 5) to the line
$3x - 4y - 12 = 0$ **3**

(b) Find the point of intersection of the lines $3x + y - 1 = 0$ and
$2x + 3y + 4 = 0$ **3**

(c) Find the point of intersection of the circle $(x-1)^2 + y^2 = 2$ and the line
$x + y + 1 = 0$ **4**

(d) Find the equation of the line parallel to the line
$3x - y - 3 = 0$ that passes through the point of intersection of
$3x + y - 1 = 0$ and $x - y + 4 = 0$ **4**

Question 3 (11 Marks) **Marks**

(a) Evaluate the following limits

(i) $\lim_{x\to 3} \frac{2x-6}{x}$ (ii) $\lim_{x\to 2} \frac{x^2-4}{x-2}$ **3**

(iii) $\lim_{x\to \infty} \frac{2x^2-3x+7}{x^2-2}$ (iv) $\lim_{x\to 3} \frac{x^2-5x+6}{x-3}$ **4**

(b) If $f(x)=x^2+2x$ differentiate $f(x)$ from first principles **4**

Question 4 (15 Marks)

Differentiate the following functions

(i) $y=3x$ (ii) $y=x^2$ **3**

(iii) $y=\frac{1}{x}$ (iv) $y=\sqrt{2x}$ **4**

(v) $y=3x^4-5x+7$ (vi) $y=3x^{\frac{3}{2}}-5x^{\frac{7}{3}}$ **4**

(vii) $y=\frac{3}{\sqrt{x}}+x^5$ (viii) $y=\frac{7}{x^3}+\sqrt[3]{x}$ **4**

Question 5 (21 Marks)

(a) Differentiate the following

(i) $y=(x^3+3x^2-9)^{10}$ (ii) $y=\frac{3x^2-5x+7}{2x^2}$ **6**

(iii) $y=(4x^2-7)(5x+4)^3$ (iv) $y=\frac{1}{\sqrt{x^3-7x}}$ **7**

(v) $y=\frac{(5-x)^3}{x^2-3}$ **4**

(b) If $f(x)=3x^2-4x+5$ evaluate

(i) $f'(x)$ (ii) $f'(0)$ **2**

(iii) $f'(2)$ (iv) Solve for x ; $f'(x)=0$ **2**

Question 6 (14 Marks) **Marks**

(a) If $y = 2x^2 - 3x + 5$

(i)Find the equation of the tangent to the curve at the point (1 , 4). **3**

(ii) Find the equation of the nomal to the curve at the point (1 , 4) **2**

(b) Find the point on the curve $y = x\sqrt{x}$ where the tangent is parallel to the line $2x - 3y + 5 = 0$ **4**

(c) Find the equations of the tangents to the curve $y = x^3 - 3x^2 + 2x - 4$ that are parallel to the line $y - 2x - 7 = 0$ **5**

YEAR 11 MATHEMATICS

TASK 4 2006.

Question 1

a) $M_{AB} = \frac{y_2-y_1}{x_2-x_1} = \frac{3+1}{-2-4}$

$= -2/3$ (2)

b) EQ^n AB

$y-y_1 = m(x-x_1)$

$y+1 = -2/3(x-4)$

$y = -2/3x + 5/3$

OR $2x+3y-5=0$ (2)

c) $D_{AB} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(-2-4)^2+(3+1)^2}$

$= \sqrt{52}$ OR $2\sqrt{13}$ (2)

d) MID PT $= M\left(\frac{-2+4}{2}, \frac{3-1}{2}\right)$

$= M(1,1)$ M

e) $m_1 \times m_2 = -1$ $m_2 = 3/2$

$y-1 = 3/2(x-1)$

$y = 3/2x - 1/2$

OR $3x-2y-1=0$ (2)

f) $\tan\theta = m$

$\tan\theta = -2/3$ $\theta = -34°$

$\theta = 146°$ OR $34°$ (2)

Question 2

a) $D_{PERP} = \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

$= \frac{|3(-1)+-4(5)-12|}{\sqrt{3^2+4^2}}$

$= \frac{35}{5} = 7$ (3)

b) $3x+y = 1$

$2x+3y = -4$

$-9x+3y = 3$ 3(A)

$2x+3y = -4$ (B)

$7x = 7$ $\therefore x = 1$ $y = -2$

$(1,-2)$ (3)

c) $(x-1)^2+y^2 = 2$ (A)

$x+y = -1$ (B)

FROM (B) $y = -(x+1)$

SUBST (A) $(x-1)^2+(-(x+1))^2 = 2$

$x^2-2x+1+x^2+2x+1 = 2$

$2x^2+2 = 2$

$2x^2 = 0$

$x = 0$ $y = -1$ $(0,-1)$ (4)

d) $3x+y = 1$ (A)

$+ \quad x-y = -4$ (B)

$4x = -3$ (A)+(B)

$x = -3/4$ $y = 13/4$

// to $3x-y-3=0$

$y = 3x-3$

gradient is 3.

EQN

$y-13/4 = 3(x+3/4)$

$y = 3x+11/2$

OR $6x-2y+11=0$ (4)

Question 3.

i) $\lim_{x\to 3} \frac{2x-6}{3} = \frac{0}{3} = 0$ (1)

ii) $\lim_{x\to 2} \frac{x^2-4}{x-2} = \lim_{x\to 2} \frac{(x+2)(x-2)}{(x-2)}$

$= \lim_{x\to 2} x+2$

$= 4$ (2)

iii) $\lim_{x\to\infty} \frac{2x^2-3x\cdot 7}{x^2-2}$

$= \lim_{x\to\infty} \frac{2-3/x+7/x^2}{1-2/x^2}$

$= \lim_{x\to\infty} \frac{2-0-0}{1-0}$

$= 2$ (2)

iv) $\lim_{x\to 3} \frac{x^2-5x+6}{x-3}$

$= \lim_{x\to 3} \frac{(x-3)(x-2)}{x-3}$

$= \lim_{x\to 3} x-2 = 1$ (2)

b) $f(x) = x^2+2x$

$f(x+h) = (x+h)^2+2(x+h)$

$= x^2+2xh+h^2+2x+h$

$\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0} \frac{x^2+2xh+h^2+2x+h-(x^2+2x)}{h}$

$= \lim_{h\to 0} \frac{2xh+h^2+2h}{h}$

$= \lim_{h\to 0} 2x+2+h$

$= 2x+2$ (4)

Question 4

i) $y = 3x$

$\frac{dy}{dx} = 3$ (1)

ii) $y = x^2$

$\frac{dy}{dx} = 2x$ (2)

iii) $y = \frac{1}{x}$

$y = x^{-1}$

$\frac{dy}{dx} = -1x^{-2}$

$= \frac{-1}{x^2}$ (2)

iv) $y = \sqrt{2x}$

$y = (2x)^{1/2}$

$\frac{dy}{dx} = \frac{1}{2}(2x)^{-1/2}\cdot 2$

$= \frac{1}{\sqrt{2x}}$ (2)

v) $y = 3x^4-5x+7$

$\frac{dy}{dx} = 12x^3-5$ (2)

vi) $y = 3x^{3/2} - 5x^{7/3}$

$\frac{dy}{dx} = 9/2x^{1/2} - 35/3x^{4/3}$ (2)

vii) $y = \frac{3}{\sqrt{x}} + x^5$

$y = 3x^{-1/2} + x^5$

$\frac{dy}{dx} = -3/2x^{-3/2} + 5x^4$

OR $= \frac{-3}{2x\sqrt{x}} + 5x^4$ (2)

viii) $y = \frac{7}{x^3} + \sqrt[3]{x}$

$y = 7x^{-3} + x^{1/3}$

$\frac{dy}{dx} = -21x^{-4} + \frac{1}{3}x^{-2/3}$

$= \frac{-21}{x^4} + \frac{1}{3\sqrt[3]{x^2}}$ (2)

Question 5.

i) $y = (x^3 + 3x^2 - 9)^{10}$

let $u = x^3 + 3x^2 - 9$

$\frac{du}{dx} = 3x^2 + 6x$

$\therefore y = u^{10}$

$\frac{dy}{du} = 10u^9$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$= 10u^9 \cdot (3x^2 + 6x)$

$= 10(3x^2 + 6x)(x^3 + 3x^2 - 9)^9$ (3)

ii) $y = \frac{3x^2 - 5x + 7}{2x^2}$

$y = 3 - 5x^{-1} + 7x^{-2}$

$\frac{dy}{dx} = 5x^{-2} + 14x^{-3}$

$= \frac{5}{x^2} - \frac{14}{x^3}$ (3)

iii) $y = (4x^2 - 7)(5x + 4)^3$

$\frac{du \cdot v}{dx} = v \cdot \frac{du}{dx} + u \cdot \frac{dv}{dx}$

let $u = 4x^2 - 7$

$\frac{du}{dx} = 8x$

let $v = (5x + 4)^3$

$\frac{dv}{dx} = 15(5x + 4)^2$

$\frac{du \cdot v}{dx} = (5x + 4)^3 \cdot 8x + (4x^2 - 7) \cdot 15(5x + 4)^2$

$= (5x + 4)^2 \{(5x + 4) 8x + 15(4x^2 - 7)\}$

$= (5x + 4)^2 (100x^2 + 32x - 105)$ (4)

(iv) $y = \frac{1}{\sqrt{x^3 - 7x}}$

$y = (x^3 - 7x)^{-1/2}$

let $u = x^3 - 7x$

$\frac{du}{dx} = 3x^2 - 7$

$y = u^{-1/2}$

$\frac{dy}{du} = -1/2\, u^{-3/2}$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$= \frac{-1}{2u^{3/2}} \cdot (3x^2 - 7)$

$= \frac{7 - 3x^2}{2(x^3 - 7x)^{3/2}}$ (3)

(v) $y = \frac{(5-x)^3}{x^2 - 3}$

let $u = (5-x)^3$

$\frac{du}{dx} = 3(5-x)^2(-1)$

$v = x^2 - 3$

$\frac{dv}{dx} = 2x$

$\frac{d\frac{u}{v}}{dx} = \frac{v \cdot \frac{du}{dx} - u \cdot \frac{dv}{dx}}{v^2}$

$= \frac{(x^2-3)(-3)(5-x)^2 - (5-x)^3 2x}{(x^2-3)^2}$

cont

$= -\frac{(5-x)^2 \{3x^2 - 9 + 10x - 2x^2\}}{(x^2-3)^2}$

$= -\frac{(5-x^2)\{x^2 + 10x - 9\}}{(x^2-3)^2}$ (4)

(b) $f(x) = 3x^2 - 4x + 5$

(i) $f'(x) = 6x - 4$ (1)

(ii) $f'(0) = -4$ (1)

(iii) $f'(2) = 12 - 4$

$= 8$ (1)

(iv) $f'(x) = 0$

$0 = 6x - 4$

$6x = 4$

$x = 2/3$ (1)

Question 6

(a) (i) $y = 2x^2 - 3x + 5$

$\frac{dy}{dx} = 4x - 3$

$\frac{dy}{dx}(x=1) = 4 - 3 = 1$

$\therefore$ EQN OF TANGENT

$y - 4 = 1(x - 1)$

$\boxed{y = x + 3}$ (3)

(ii) EQN OF NORMAL

$y - 4 = -1(x - 1)$

$\boxed{y = -x + 5}$ (2)

(b) $y = x\sqrt{x}$

$y = x^{3/2}$

$\frac{dy}{dx} = 3/2\, x^{1/2}$

NOW $2x - 3y + 5 = 0$ r4

$3y = 2x + 5$

$y = 2/3\, x + 5/3$

$\therefore$ gradient $= 2/3$

$\therefore 3/2\sqrt{x} = 2/3$

$\sqrt{x} = 4/9$

$x = 16/81$

PT $(16/81, 64/729)$ (4)

(c) $y = x^3 - 3x^2 + 2x - 4$

$\frac{dy}{dx} = 3x^2 - 6x + 2$

NOW $y = 2x + 7$ HAS GRADIENT 2

$\therefore 2 = 3x^2 - 6x + 2$

$0 = 3x^2 - 6x$

$0 = 3x(x - 2)$

$\therefore x = 0, 2$

$y = -4, -4$

TANG 1

$y + 4 = 2(x - 0)$

$\boxed{y = 2x - 4}$

TANG 2

$y + 4 = 2(x - 2)$

$\boxed{y = 2x - 8}$ (5)