

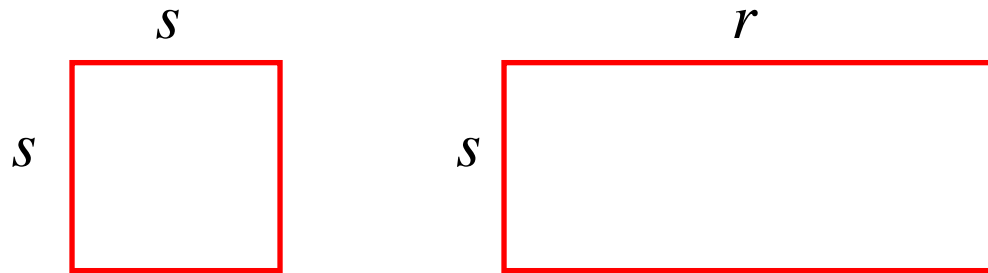
Maxima & Minima Problems

When solving word problems we need to;

- (1) Reduce the problem to a set of equations
(there will usually be two)
 - a) one will be what you are maximising/minimising
 - b) one will be some information given in the problem
- (2) Rewrite the equation we are trying to minimise/maximise with only one variable
- (3) Use calculus to solve the problem

e.g. A rope 36 metres in length is cut into two pieces. The first is bent to form a square, the other forms a rectangle with one side the same length as the square.

Find the length of the square if the sum of the areas is to be a maximum.



$$A = s^2 + rs \dots (1)$$

$$6s + 2r = 36 \dots (2)$$

make r the subject in (2)

$$2r = 36 - 6s$$

$$r = 18 - 3s$$

substitute into (1)

$$A = s^2 + (18 - 3s)s$$

$$A = s^2 + 18s - 3s^2$$

$$A = 18s - 2s^2$$

$$A = 18s - 2s^2$$

$$\frac{dA}{ds} = 18 - 4s$$

$$\frac{d^2A}{ds^2} = -4$$

Stationary points occur when $\frac{dA}{ds} = 0$

$$\text{i.e. } 18 - 4s = 0$$

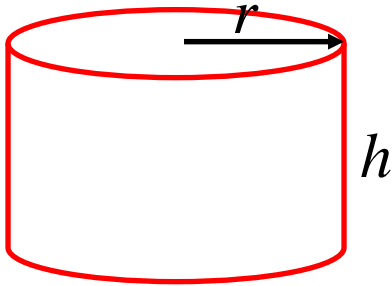
$$s = \frac{9}{2}$$

$$\text{when } s = \frac{9}{2}, \frac{d^2A}{ds^2} = -4 < 0$$

\therefore when the side length of the square is $4\frac{1}{2}$ metres, the area is a maximum

(ii) An open cylindrical bucket is to be made so as to hold the largest possible volume of liquid.

If the total surface area of the bucket is 12π units², find the height, radius and volume.



$$V = \pi r^2 h \dots (1)$$

$$12\pi = \pi r^2 + 2\pi r h \dots (2)$$

make h the subject in (2)

$$2\pi r h = 12\pi - \pi r^2$$

$$h = \frac{12 - r^2}{2r}$$

substitute into (1)

$$V = \pi r^2 \left(\frac{12 - r^2}{2r} \right)$$

$$V = 6\pi r - \frac{1}{2}\pi r^3$$

$$V = 6\pi r - \frac{1}{2}\pi r^3$$

$$\frac{dV}{dr} = 6\pi - \frac{3}{2}\pi r^2$$

$$\frac{d^2V}{dr^2} = -3\pi r$$

Stationary points occur when $\frac{dV}{dr} = 0$

$$\text{i.e. } 6\pi - \frac{3}{2}\pi r^2 = 0$$

$$\frac{3}{2}\pi r^2 = 6\pi$$

$$r^2 = 4$$

$$r = \pm 2$$

$$\text{when } r = 2, \frac{d^2V}{dr^2} = -6\pi < 0$$

\therefore when $r = 2$, V is a maximum

$$\begin{aligned} \text{when } r = 2, h &= \frac{12 - 2^2}{2(2)} \\ &= 2 \end{aligned}$$

$$\begin{aligned} V &= \pi(2)^2(2) \\ &= 8\pi \end{aligned}$$

\therefore maximum volume is 8π units³ when the radius and height are both 2 units

Exercise 10H; evens (not 30)

Exercise 10I; evens