# *Probability & Counting Techniques*

2007 Extension 1 HSC Q5b)

Mr and Mrs Roberts and their four children go to the theatre. They are randomly allocated six adjacent seats in a single row.

What is the probability that the four children are allocated seats next to each other?

ways of arranging 3 objects

i.e 2 adults + 1 group of 4 children

ways of arranging 4 children

$$P(\text{children sit next to each other}) = \frac{3!4!}{6!}$$

ways of arranging 6 people

$$= \frac{1}{5}$$

2007 Extension 2 HSC Q5a)

A bag contains 12 red marbles and 12 yellow marbles. Six marbles are selected at random without replacement.

(i) Calculate the probability that exactly three of the selected marbles are red. Give your answer correct to two decimal places.

$$P(3 \text{ red}) = \frac{{}^{12}C_3 \times {}^{12}C_3}{{}^{24}C_6}$$

$$= 0.3595\ldots$$

$$= 0.36 \quad \text{(to 2 dp)}$$

(ii) Hence, or otherwise, calculate the probability that more than three of the selected marbles are red. Give your answer correct to two decimal places.

$$P(> 3 \text{ red}) = P(4 \text{ red}) + P(5 \text{ red}) + P(6 \text{ red})$$

$$= \frac{{}^{12}C_4 \times {}^{12}C_2 + {}^{12}C_5 \times {}^{12}C_1 + {}^{12}C_6 \times {}^{12}C_0}{{}^{24}C_6}$$

$$= 0.3202\ldots$$

$$= 0.32 \quad \text{(to 2 dp)}$$

***OR***

$$P(>3 \text{ red}) = 1 - P(3 \text{ red}) - P(<3 \text{ red})$$

$$= 1 - P(3 \text{ red}) - P(>3 \text{ yellow})$$

$$= 1 - P(3 \text{ red}) - P(>3 \text{ red})$$

$$2P(>3 \text{ red}) = 1 - P(3 \text{ red})$$

$$P(>3 \text{ red}) = \frac{1}{2}\{1 - P(3 \text{ red})\}$$

$$= \frac{1}{2}(1 - 0.3595\ldots)$$

$$= 0.3202\ldots$$

$$= 0.32 \quad \text{(to 2 dp)}$$

2006 Extension 2 HSC Q5d)

In a chess match between the Home team and the Away team, a game is played on board 1, board 2, board 3 and board 4.

On each board, the probability that the Home team wins is 0.2, the probability of a draw is 0.6 and the probability that the Home team loses is 0.2.

The results are recorded by listing the outcomes of the games for the Home team in board order. For example, if the Home team wins on board 2, draws on board 2, loses on board 3 and draws on board 4, the result is recorded as WDLD.

(i) How many different recordings are possible?

$$\text{Recordings} = 3 \times 3 \times 3 \times 3$$

$$= 81$$

(ii) Calculate the probability of the result which is recorded as WDLD.

$$P(\text{WDLD}) = 0.2 \times 0.6 \times 0.2 \times 0.6$$

$$= 0.144$$

(iii) Teams score 1 point for each game won, $\frac{1}{2}$ a point for each game drawn and 0 points for each game lost.

What is the probability that the Home team scores more points than the Away team?

***first calculate probability of equal points***

$$P(4 \text{ draws}) = 0.6^4$$

$$= 0.1296$$

ways of arranging WWLL

$$P(2 \text{ wins, } 2 \text{ losses}) = 0.2^2 \times 0.2^2 \times \frac{4!}{2!2!}$$

$$= 0.0096$$

ways of arranging WLDD

$$P(1 \text{ win, } 1 \text{ loss, } 2 \text{ draws}) = 0.2 \times 0.2 \times 0.6^2 \times \frac{4!}{2!}$$

$$= 0.1728$$

$$P(\text{equal points}) = 0.1296 + 0.0096 + 0.1728$$

$$= 0.312$$

$$P(\text{unequal points}) = 1 - 0.312$$

$$= 0.688$$

As the probabilities are equally likely for the Home and Away teams, then either the Home team has more points or the Away team has more points.

$$P(\text{Home team more points}) = \frac{1}{2} P(\text{unequal points})$$

$$= \frac{1}{2} \times 0.688$$

$$= 0.344$$

2002 Extension 2 HSC Q4c)

From a pack of nine cards numbered 1, 2, 3, …, 9, three cards are drawn at random and laid on a table from left to right.

(i) What is the probability that the number exceeds 400?

$$P(>400) = \frac{6}{9}$$

$$= \frac{2}{3}$$

*it is the same as saying; "what is the probability of the first number being >4"*

(ii) What is the probability that the digits are drawn in descending order?

total arrangements of 3 digits $= 3!$

$$= 6$$

Only one arrangement will be in descending order

$$P(\text{descending order}) = \frac{1}{6}$$

**Exercise 10H; odd**