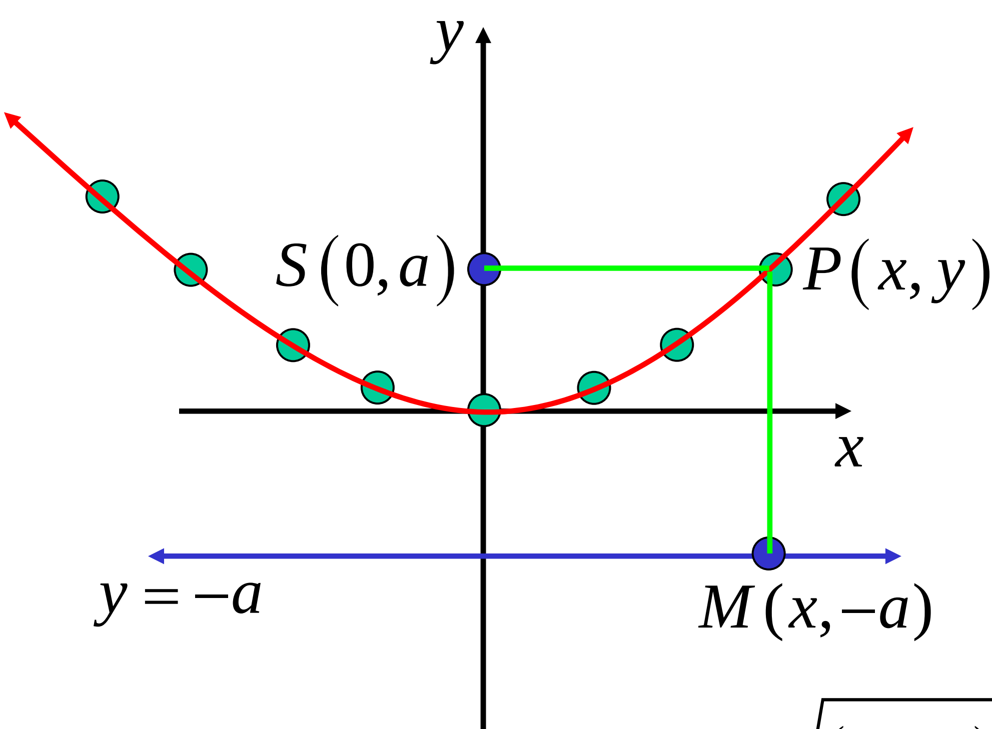
The Parabola As a Locus



A point moves so that its distance from a fixed point (**focus**) is equal to its distance from a fixed line (**directrix**)

$$d_{PS} = d_{PM}$$

$$\sqrt{(x-0)^2+(y-a)^2} = \sqrt{(x-x)^2+(y+a)^2}$$

$$x^2+(y-a)^2 = (y+a)^2$$

$$x^2+y^2-2ay+a^2 = y^2+2ay+a^2$$

$$x^2 = 4ay$$

$$x^2 = 4ay$$

vertex: $(0,0)$

focus: $(0,a)$

directrix: $y = -a$

focal length: a units

e.g. (i) Find the focus, focal length and directrix;

a) $x^2 = 32y$

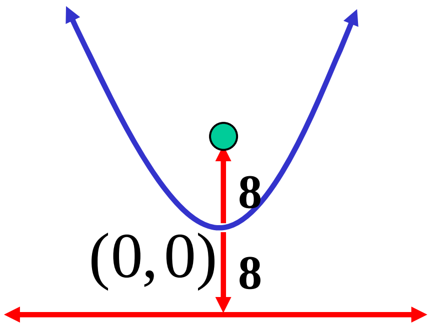
$4a = 32$

$a = 8$

focal length = 8 units

focus is $(0,8)$

directrix is $y = -8$



b) $y = 4x^2 \Rightarrow x^2 = \frac{1}{4}y$

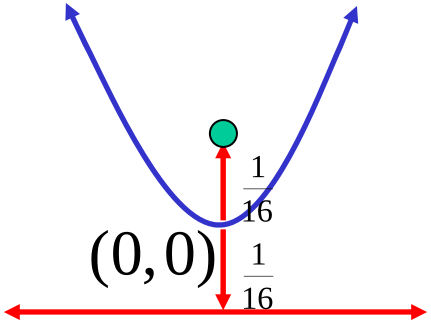
$4a = \frac{1}{4}$

$a = \frac{1}{16}$

focus is $\left(0, \frac{1}{16}\right)$

directrix is $y = -\frac{1}{16}$

focal length = $\frac{1}{16}$ unit



(ii) Find the equation of the parabola with;

a) focus $(0,-2)$, directrix $y=2$

$a=-2 \qquad x^2=4(-2)y$

$x^2=-8y$

b) focus $(3,0)$, directrix $x=-3$

$a=3 \qquad y^2=4(3)x$

$y^2=12x$

Vertex NOT at the origin

$$(x-p)^2=4a(y-q)$$

vertex: (p,q)

focus: $(p,q+a)$

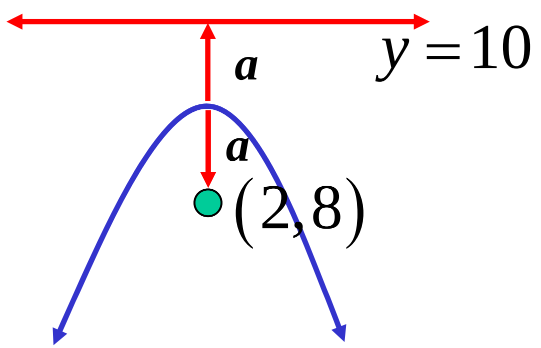
directrix: $y=q-a$

focal length: a units

e.g. (i) Find the equation of the parabola with vertex $(3,1)$ and focal length 2 units

$$(x-3)^2 = 4(2)(y-1)$$
$$(x-3)^2 = 8(y-1)$$
$$x^2 - 6x + 9 = 8y - 8$$
$$8y = x^2 - 6x + 17$$
$$y = \frac{1}{8}\left(x^2 - 6x + 17\right)$$

(ii) focus (2,8) and directrix $y = 10$



$$2a = 2$$
$$a = 1 \quad \text{vertex is } (2,9)$$

$$(x-2)^2 = -4(1)(y-9)$$
$$(x-2)^2 = -4(y-9)$$
$$x^2 - 4x + 16 = -4y + 36$$
$$4y = -x^2 + 4x + 20$$
$$y = -\frac{1}{4}\left(x^2 - 4x - 20\right)$$

(iii) Find the vertex, focus, focal length, directrix of $12y = x^2 - 6x - 3$

$$12y = x^2 - 6x - 3$$

$$12y + 3 = x^2 - 6x$$

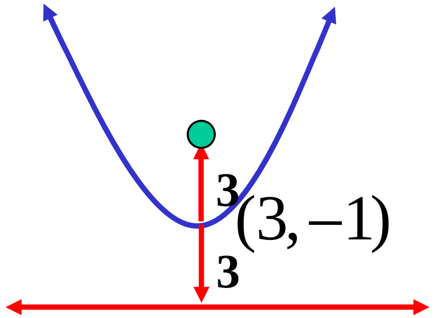
$$12y + 3 + 9 = (x-3)^2$$

$$12y + 12 = (x-3)^2$$

$$12(y+1) = (x-3)^2$$

$$4a = 12$$
$$a = 3$$

vertex: $(3, -1)$



$\therefore$ focal length = 3 units

vertex = $(3, -1)$

focus = $(3, 2)$

directrix: $y = -4$

Exercise 9B; *1,2 try at home*
4 *(use definition)*
6ace etc, 7ac, 8ace, 9ace, 10ac, 11bd, 12a

Exercise 9C; 3 to 8 ace etc, 10ac, 11ace, 12