

<p>Example</p> <p>Prove that</p> <p>$5^n - 1$ is divisible by 4 (n a positive integer).</p>	<p>Prove that</p> <p>$3^{2n} - 1$ is divisible by 8 (n a positive integer).</p>
<p>Step 1</p> <p>Testing $n = 1$, Expression = $5^1 - 1$ = 4 (which is divisible by 4) \therefore the result is true for $n = 1$.</p>	<p>Step 1</p>
<p>Step 2</p> <p>Assume the result is true for $n = k$, that is, assume $5^k - 1 = 4 \times P$, where P is an integer.</p>	<p>Step 2</p>
<p>Step 3</p> <p>Hence show for $n = k + 1$, that $5^{k+1} - 1 = 4 \times Q$ where Q is an integer.</p> <p>Now</p> $5^{k+1} - 1 = 5^k \times 5^1 - 1$ $= 5 \times (4P + 1) - 1, \text{ from our assumption,}$ $= 20P + 4$ $= 4(5P + 1)$ $= 4 \times Q \text{ since } 5P + 1 \text{ is integral.}$ <p>Hence, if the result is true for $n = k$, then it is true for $n = k + 1$.</p>	<p>Step 3</p>
<p>Step 4</p> <p>Since the result is true for $n = 1$, from Step 3 it is true for $n = 1 + 1 = 2$, and then for $n = 2 + 1 = 3$, and so on for all positive integral values of n.</p>	<p>Step 4</p>
<p>Exercise:</p> <p>Prove:</p> <p>If n is even, then $n^2 + 2n$ is divisible by 8.</p>	