

<p>Example</p> <p>Show</p> $3^n \geq 1 + 2n.$	<p>Prove that</p> $5^n \geq 1 + 4n.$
<p>Step 1</p> <p>Testing $n = 1$,</p> <p>LHS = 3</p> <p>RHS = $1 + 2 \times 1 = 3$,</p> <p>\therefore the result is true for $n = 1$.</p>	<p>Step 1</p>
<p>Step 2</p> <p>Assume the result is true for $n = k$, that is,</p> $3^k \geq 1 + 2k \quad (1)$	<p>Step 2</p>
<p>Step 3</p> <p>Hence show the result is true for $n = k + 1$, that is,</p> <p>show $3^{k+1} \geq 1 + 2(k + 1)$.</p> $3^{k+1} \geq 2k + 3$ <p>Now $3^{k+1} = 3^1 \cdot 3^k$</p> $\geq 3(1 + 2k) \text{ using (1),}$ <p>$\therefore 3^{k+1} \geq 3 + 6k$,</p> <p>that is, $3^{k+1} \geq 2k + 3 + 4k$</p> $> 2k + 3,$ <p>since k is a positive integer.</p> <p>Hence, if the result is true for $n = k$, it is true for $n = k + 1$.</p>	<p>Step 3</p>
<p>Step 4</p> <p>Since the result is true for $n = 1$, from Step 3 it is true for $n = 1 + 1 = 2$, and then for $n = 2 + 1 = 3$, and so on for all positive integral values of n.</p>	<p>Step 4</p>
<p>Exercise:</p> <p>Show:</p> $(1 + p)^n \geq 1 + np \text{ where } p > -1.$	