Question 1 (25 marks) **Marks**

(a) The lines $3x - y + 2 = 0$ and $mx - y - 1 = 0$ intersect at 45°.
Find the possible value(s) of m. **3**

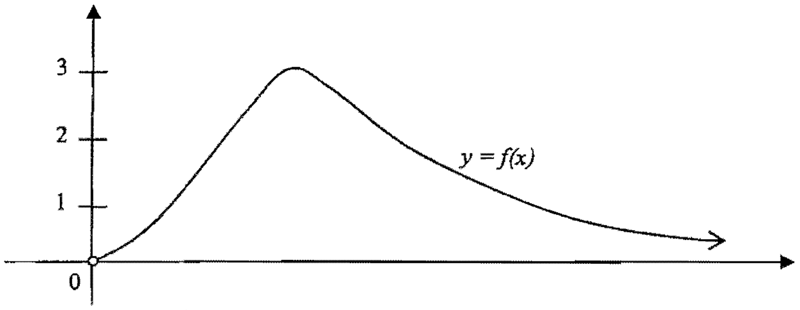
(b) Solve $\dfrac{1}{7-3x} < 1$, graphing your solution on a number line. **3**

(c) Simplify $\dfrac{1}{3\sqrt{2}+1} + \dfrac{1}{1-3\sqrt{2}}$ giving your answer as a single fraction with a rational denominator. **3**

(d) Factorise fully $(k-1)^3 + (k+2)^3$ **3**

(e) Find the equation of the tangent to the curve $y = 2x\sqrt{x+1}$ at the point where $x = 3$. **4**

(f) Consider the graph of $y = f(x)$



(i) State the domain. **1**
(ii) State the range. **2**
(iii) Write the equation of any asymptotes. **1**

(g) Find the coordinates of the point P which divides the interval AB with end points A (2,3) and B (5,7) internally in the ratio 4:9. **3**

(h) Simplify fully $\dfrac{3^{x+1} + 3^{x-1}}{3^x}$ **2**

Question 2 (27marks) **Marks**

omit

(a) The polynomial, $P(x) = x^4 + 6x^3 + 5x^2 - 12x$.

(i) Find $P(-4)$. **1**

(ii) Hence or otherwise find the factors of $P(x)$. **3**

(iii) Draw a neat sketch of $y = P(x)$ in the domain $-5 \le x \le 2$. **3**

(iv) Hence, or otherwise, solve $x^4 + 6x^3 + 5x^2 - 12x \le 0$. **2**

(b) Find the value of *p* if $x^4 + x^3 - px^2 - x + 6$ is exactly divisible by $(x+1)$. **2**

(c) If α, β and γ are the roots of the equation $x^3 - 3x^2 + 4x - 6 = 0$, find the value of

(i) $\alpha + \beta + \gamma$ (ii) $\alpha\beta + \beta\gamma + \alpha\gamma$ (iii) $\alpha\beta\gamma$ **3**

(iv) $\alpha^2 + \beta^2 + \gamma^2$ (v) $(\alpha+2)(\beta+2)(\gamma+2)$ **6**

(d) Find the value of *k* if the equation $2x^3 + 3x^2 + kx - 48 = 0$ has two roots equal in value but opposite in sign. **4**

(e) Divide the polynomial $P(x) = x^3 + 5x^2 - 22x - 6$ by $D(x) = x^2 - 3x + 2$. Hence write in the form $P(x) = D(x).q(x) + r(x)$. **3**

Question 3 (20 marks) **Marks**

(a) Solve $2\cos(2\theta - 30^\circ) = \sqrt{3}$ $\quad 0^\circ \le \theta \le 360^\circ$ **3**

(b) Solve $2\cos^2 x - \sin x = 2$, $\quad 0^\circ \le x \le 360^\circ$. **4**

(c) Show that $(\sec A + \tan A)^2 = \dfrac{1 + \sin A}{1 - \sin A}$ **4**

(d) Express $\sqrt{3}\cos x - \sin x$ in the form $A\cos(x + \phi)$.
Hence solve $\sqrt{3}\cos x - \sin x = 1$, $0^\circ \le x \le 360^\circ$. **4**

(e) Solve $3\sin\theta + \cos\theta = 2$ by using $t = \tan\left(\dfrac{\theta}{2}\right)$, $0^\circ \le x \le 360^\circ$. **5**

Question 4 (22 marks) **Marks**

omit

(a) How many numbers greater than 3000 can be formed from the digits 1, 3, 5, 7, 9 if no repetitions are permitted? **3**

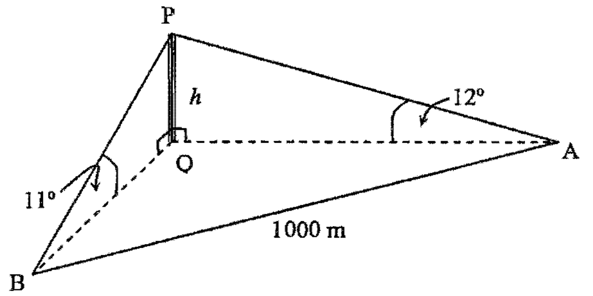
(b) Mr and Mrs Jones and 6 guests sit around the dinner table. In how many ways can thay be arranged if the two hosts are separated? **3**

(c) In how many ways can the letters of the word DEDUCED be arranged? **2**

(d) In how many ways can 5 different Mathematics books , 4 different Physics books and 2 different Chemistry books be arranged on a shelf if the books in each subject are to be together. **3**

(f) In how many ways can a committee of 3 women and 4 men be chosen from 8 women and 7 men if two particular women refuse to serve on the committee together? **4**

(g)



The angle of elevation of a tower PQ of height h metres at appoint A due east of it is 12°. From another point B, the bearing of the tower is 051°T and the angle of elevation is 11°. The points A and B are 1000 metres apart and on the same level as the base Q of the tower.

(i) Show that $\angle AQB = 141^\circ$. **1**

(ii) Consider the triangle APQ and show that $AQ = h\tan 78^\circ$ **2**

(iii) Find a similar expression for BQ. **1**

(iv) Use the cosine rule in the triangle AQB to calculate h to the nearest metre. **3**

Question 5 (24 marks) **Marks**

(a) Write the set of parametric equations for $x^2 = 20y$ 2

(b) Find the Cartesian equation of $x = t - 1$ and $y = t^2 + t$ 2

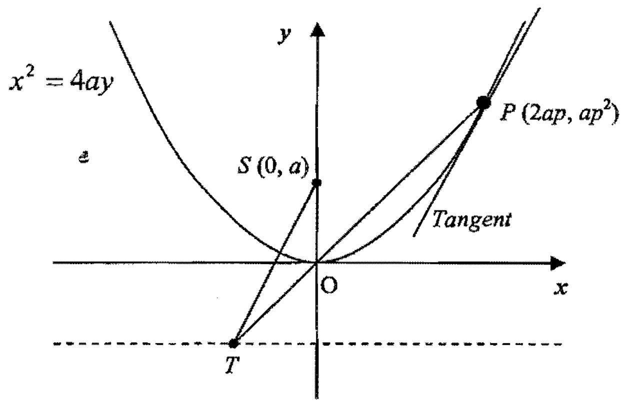
(c) (i) Find the equation of the tangent to the curve $x^2 = -2y$ at the point $(4, -8)$. 3

(ii) This tangent meets the directrix at point M. Find the coordinates of M 2

(d) (i) Find the coordinates of Q on the parabola $x = 8t$, $y = 4t^2$ at the point where $t = -1$. 2

(ii) Find the equation of the normal to the parabola at Q. 3

(e)



The straight line drawn from a point $P\ (2ap, ap^2)$, on a parabola to the vertex cuts the directrix at T. Prove that if the focus is S, that TS is parallel to the tangent at P. 5

(f) The points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ are two points on the parabola $x^2 = 4ay$. The tangents at P and Q intersect at T. Given that the equation of the tangent is $y = px - ap^2$.

(i) Show that T has the coordinates $(a(p+q), apq)$. 2

(ii) Hence, show that if the pararmeter $q = p - 1$, the locus of T is

$$x^2 = 4a\left(y + \frac{a}{4}\right)$$ 3

Year 11 – YEARLY (2006)
Extension 1 – Mathematics.

Q1 a) $3x - y - 1 = 0$ $\quad$ $mx - y - 1 = 0$

$y = 3x - 1$ $\quad$ $y = mx - 1$

$\tan 45^\circ = \left|\dfrac{3-m}{1+3m}\right|$

$1 = \left|\dfrac{3-m}{1+3m}\right|$

$\therefore 1+3m = -(3-m)$ & $1+3m = 3-m$

$2m = -4$ $\quad$ $4m = 2$

$\underline{m = -2}$ $\quad$ $\underline{m = \frac{1}{2}}$

3

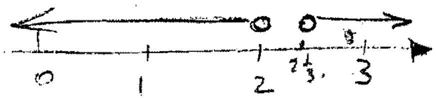
b) $\dfrac{1}{7-3x} < 1$

$\therefore 7 - 3x \neq 0$ $\quad$ $\dfrac{1}{7-3x} = 1$

$3x \neq 7$ $\quad$ $1 = 7 - 3x$

$x \neq \frac{7}{3}.$ $\quad$ $3x = 6.$

$x \neq 2\frac{1}{3}.$ $\quad$ $\underline{x = 2}$



Test $\underline{x=0}$

$\dfrac{1}{7-3(0)} < 1$

$\frac{1}{7} < 1$ $\quad$ ✓ TRUE

$\therefore$ $\underline{x < 2 \ , \ x > 2\frac{1}{3}}$

3

c) $\dfrac{1}{3\sqrt{2}+1} + \dfrac{1}{1-3\sqrt{2}}$

$= \dfrac{3\sqrt{2}-1}{18-1} + \dfrac{1+3\sqrt{2}}{1-18}$

$= \dfrac{3\sqrt{2}-1}{17} - \dfrac{(1+3\sqrt{2})}{17}$

$= \underline{\dfrac{-2}{17}}$

3

d) $(k-1)^3 + (k+2)^3$

$= (k-1+k+2)\left((k-1)^2 - (k-1)(k+2) + (k+2)^2\right)$

$= (2k+1)\left(k^2 - 2k + 1 - (k^2 + k - 2) + k^2 + 4k + 4\right)$

$= (2k+1)(k^2 + k + 7)$

3

(e) $y = 2x\sqrt{x+1}$

$\dfrac{dy}{dx} = \sqrt{x+1} \cdot 2 + 2x \cdot \frac{1}{2}(x+1)^{-\frac{1}{2}} \times 1.$

$= 2\sqrt{x+1} + \dfrac{x}{\sqrt{x+1}}$

$= \dfrac{2(x+1) + x}{\sqrt{x+1}}$

$\dfrac{dy}{dx} = \dfrac{3x+2}{\sqrt{x+1}}$

when $\underline{x=3.}$ $\rightarrow$ $\underline{y=12}$

& $\dfrac{dy}{dx} = \dfrac{11}{2}$

$\therefore$ EQN $y - 12 = \frac{11}{2}(x-3)$

$2y - 24 = 11x - 33$

4

$\underline{11x - 2y - 9 = 0.}$ $\quad$ or $\underline{y = \frac{11}{2}x - \frac{9}{2}.}$

f) i) D: $x > 0$ $\quad$ 1

ii) R: $0 < y \leq 3$. $\quad$ 2

iii) Asymptote $\underline{y=0}$ $\quad$ 1

g) i) $A(2,3)$ $\quad$ $B(5,7)$

$4 : 9$

$x = \dfrac{9(2) + 4(5)}{4+9}$ $\quad$ $y = \dfrac{9(3) + 4(7)}{4+9}$

$= \dfrac{18+20}{13}$ $\quad$ $= \dfrac{27+28}{13}$

$= \dfrac{38}{13}$ $\quad$ $= \dfrac{55}{13}$

$\underline{x = 2\frac{12}{13}}$ $\quad$ $\underline{y = 4\,^3\!/_{13}}$

3

h) $\dfrac{3^{x+1} + 3^{x-1}}{3^x}$ $\quad$ OR $= \dfrac{3^x(3^1 + 3^{-1})}{3^x}$

$= 3^1 + 3^{-1}$

$= 3 + \frac{1}{3}$ $\quad$ $= 3 + \frac{1}{3}.$

$= 3\frac{1}{3}.$ $\quad$ $= 3\frac{1}{3}.$

2

Q3 a) $\cos(2\theta - 30^\circ) = \frac{\sqrt{3}}{2}$ $0^\circ \le 2\theta \le 720^\circ$

$-30^\circ \le 2\theta - 30 \le 690^\circ$

$2\theta - 30^\circ = -30^\circ, 30^\circ, 330^\circ, 390^\circ, 690^\circ$

$2\theta = 0^\circ, 60^\circ, 360^\circ, 420^\circ, 720^\circ.$

$\theta = 0^\circ, 30^\circ, 180^\circ, 210^\circ, 360^\circ$ (3)

b) $2(1 - \sin^2 x) - \sin x - 2 = 0$

$-2\sin^2 x - \sin x = 0$

$\sin x\,(2\sin x + 1) = 0$

$\therefore \sin x = 0$ $\sin x = -\frac{1}{2}.$

$\therefore x = 0, 180^\circ, 360^\circ$ $x = 210^\circ, 330^\circ$ (4)

c) LHS $= \sec^2 A + 2\sec A \tan A + \tan^2 A$

$= 1 + \tan^2 A + \frac{2\sin A}{\cos^2 A} + \tan^2 A$

$= 1 + \frac{2\sin^2 A}{\cos^2 A} + \frac{2\sin A}{\cos^2 A}$

$= \frac{\cos^2 A + 2\sin^2 A + 2\sin A}{\cos^2 A}$

$= \frac{\cos^2 A + \sin^2 A + \sin^2 A + 2\sin A}{1 - \sin^2 A}$

$= \frac{1 + 2\sin A + \sin^2 A}{(1 - \sin A)(1 + \sin A)}$

$= \frac{(1 + \sin A)\cancel{(1 + \sin A)}}{(1 - \sin A)\cancel{(1 + \sin A)}}$

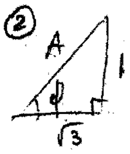
$=$ RHS. (4)

d) $\sqrt{3}\cos x - \sin x = A\cos(x + \phi)$

$= A\cos x \cos\phi - A\sin x \sin\phi$

$\therefore \sqrt{3} = A\cos\phi$ $-1 = -A\sin\phi$ 2

$\cos\phi = \frac{\sqrt{3}}{A}$ (1) $\sin\phi = \frac{1}{A}$ (2)



By Squaring & Adding (1) & (2)

$A^2 = 3 + 1$; $\tan\phi = \frac{1}{\sqrt{3}}.$

$\underline{A = 2}$ $\underline{\phi = 30^\circ}$

$\therefore \sqrt{3}\cos x - \sin x = 2\cos(x + 30^\circ)$

d) (cont.)

$\therefore$ Solving $\sqrt{3}\cos x - \sin x = 1$

$2\cos(x + 30^\circ) = 1$

$\cos(x + 30^\circ) = \frac{1}{2}$

$x + 30^\circ = 60^\circ, 300^\circ$

$x = 30^\circ, 270^\circ$ (4)

e) $3\sin\theta + \cos\theta = 2$ $t = \tan\frac{\theta}{2}.$

$3\left(\frac{2t}{1+t^2}\right) + \left(\frac{1-t^2}{1+t^2}\right) = 2$

$6t + 1 - t^2 = 2 + 2t^2$

$3t^2 - 6t + 1 = 0$

$t = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(1)}}{2(3)}$

$= \frac{6 \pm \sqrt{24}}{6.}$

$t = \frac{3 \pm \sqrt{6}}{3.}$ $0 \le \theta \le 360$

$\tan\frac{\theta}{2} = \frac{3 \pm \sqrt{6}}{3.}$ $\therefore 0 \le \frac{\theta}{2} \le 180^\circ$

$\frac{\theta}{2} = 61^\circ 10', 10^\circ 24'$ (5)

$\theta = 20^\circ 48', 122^\circ 20'$ Test $\underline{\theta = 180^\circ}$ Not a solution

Q4 a) (5 digits) $= 5! = 120$

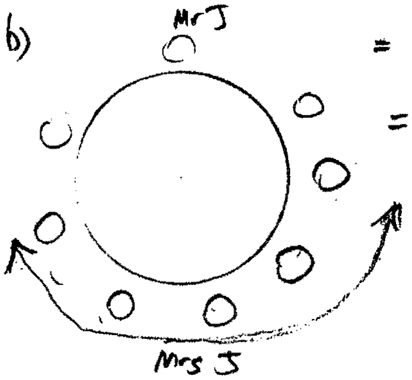
(4 digits) $= 4 \times (4 \times 3 \times 2) = 96$

TOTAL $= 120 + 96$

$= \underline{216}$ numbers

3

b)



$= 1 \times 5 \times 6!$

$= \underline{3600}$ ways.

3

c) DEDUCED $= \dfrac{7!}{3!\,2!}$ D's → 3!, 2! ← E's

$= \underline{420}$

2

d) Maths Physics Chem.

$= 3! \times (5! \times 4! \times 2!)$

Groups ↗   Within Groups ↗

$= \underline{103\,680}$ ways

3

f) ${}^7C_4 \times {}^6C_2 \times 2 + {}^7C_4 \times {}^6C_3$

(Men) (Women) — 1 particular woman chosen & 1 particular women not chosen

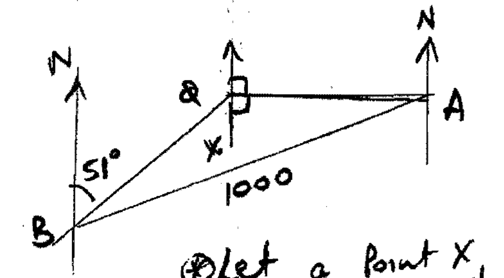
(Men) (Women) — 2 particular women not chosen.

$= 1050 + 700$

$= \underline{1750}$

4

g)



Let a point X be due South of Q.

i) $\angle BQX = 51°$ (alt ∠'s in // lines)

$\therefore \angle AQB = 90 + 51°$

$\underline{\angle AQB = 141°}$

1

ii) $\angle APQ = 78°$ (angle sum of Δ

$\therefore \tan 78 = \dfrac{AQ}{h}$

$\underline{AQ = h \tan 78°}$

2

iii) $\therefore \underline{BQ = h \tan 79°}$

1

iv) Using the cosine rule

$AB^2 = AQ^2 + BQ^2 - 2(AQ)(BQ)\cos 141°$

$(1000)^2 = (h\tan 78)^2 + (h\tan 79)^2 - 2(h\tan 78)(h\tan 79)\cos 14$

$1000^2 = h^2[\tan^2 78 + \tan^2 79 - 2\tan 78 \tan 79 \cos 141]$

$h^2 = \dfrac{1000^2}{\tan^2 78 + \tan^2 79 - 2\tan 78 \tan 79 \cos}$

$h = \dfrac{1000}{\sqrt{(\tan^2 78 + \tan^2 79 - 2\tan 78 \tan 79 \cos)4}}$

$h = 107.696$ m.

$\underline{h \doteqdot 108 \text{ metres}}$

3

Q5 a) $x^2 = 20y$

$\therefore 20 = 4a$

$a = 5$

$\therefore x = 2at \quad , y = at^2$

$\underline{x = 10t} \quad , \underline{y = 5t^2}$ (2)

b) $x = t - 1$ ① $, y = t^2 + t$ ②

$\therefore t = x + 1$ ③

sub ③ into ②

$y = (x+1)^2 + (x+1)$

$= x^2 + 2x + 1 + x + 1$

$\underline{y = x^2 + 3x + 2}$ (2)

c) i) $x^2 = -2y \qquad (4, -8)$

$y = -\frac{1}{2}x^2$

$\frac{dy}{dx} = -x$

when $x = 4$ $\quad \frac{dy}{dx} = -4$

$\therefore y - (-8) = -4(x - 4)$

$y + 8 = -4x + 16$

$\underline{y = -4x + 8}$ (3)

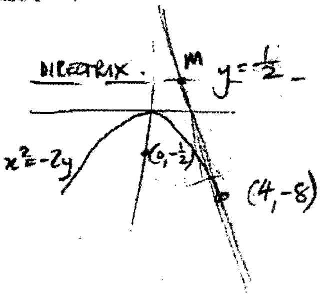
ii) since $x^2 = -2y$.

$\therefore -2 = -4a$

$a = \frac{1}{2}$

$\therefore$ DIRECTRIX is

$\underline{y = \frac{1}{2}}$



when $y = \frac{1}{2}$ $\quad \frac{1}{2} = -4x + 8$

$4x = 7\frac{1}{2} = \frac{15}{2}$

$\underline{x = \frac{15}{8} = 1\frac{7}{8}}. \quad \therefore M\left(\frac{15}{8}, \frac{1}{2}\right)$ (2)

d) i) $x = 8t \quad , y = 4t^2$

when $t = -1$ $\quad x = 8(-1) , y = 4(-1)^2$

$\underline{x = -8} \quad , \underline{y = 4}$ (2)

d) ii) $\therefore x = 8t \qquad y = 4t^2$

$\frac{dx}{dt} = 8 \qquad \frac{dy}{dt} = 8t$

$\frac{dy}{dx} = \frac{8t}{8}$

$\frac{dy}{dx} = t$

when $t = -1$ $\quad \underline{\frac{dy}{dx} = -1}$ (gradient of Tangent) $\therefore$ Gradient of Normal $= 1$

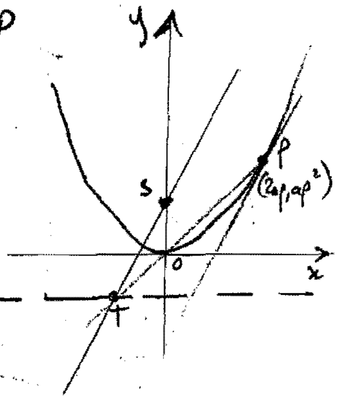
$\therefore$ EQN of NORMAL where $m = 1$ @ $(-8, 4)$

$y - 4 = 1(x - (-8))$

$y - 4 = x + 8$

$\underline{y = x + 12}$ (3)

e) Gradient of Tangent @ P



$x = 2ap \qquad y = ap^2$

$\frac{dx}{dp} = 2a \qquad \frac{dy}{dp} = 2ap$.

$\frac{dy}{dx} = \frac{2ap}{2a} = p$.

Gradient of Line PT (or PO)

$m_{PO} = \frac{ap^2 - 0}{2ap - 0}$

$m_{PO} = \frac{p}{2} \qquad \therefore$ Eqn of PO @ $(0,0)$

$\underline{y = \frac{p}{2}x}$.

Eqn of Directrix $\underline{y = -a}$

$\therefore$ Coords of T

Solving simultaneously.

$-a = \frac{p}{2}x$

$x = \frac{-2a}{p}$. $\qquad \therefore T\left(\frac{-2a}{p}, -a\right)$

$\therefore$ Gradient of TS. where $S(0, a)$

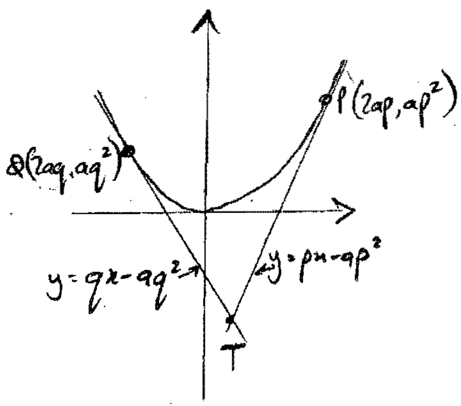
$m_{TS} = \frac{a - (-a)}{0 - \left(-\frac{2a}{p}\right)}$

$= \frac{2a}{2a/p}$ (5)

$m_{TS} = p$. $\qquad \therefore \boxed{m_{TS} = m_{\text{Tangent at } P}}$.

$\therefore$ TS is parallel to the tangent at P.

Q5) i) $P(2ap, ap^2)$
$Q(2aq, aq^2)$



$\therefore$ Eqn of tangent at P is

$\underline{y = px - ap^2}$ (1)

$\therefore$ Eqn of tangent at Q is

$\underline{y = qx - aq^2}$ (2)

Solving simultaneously.

$px - ap^2 = qx - aq^2$

$px - qx = ap^2 - aq^2$

$x(p-q) = a(p^2 - q^2)$

$x\cancel{(p-q)} = a\cancel{(p-q)}(p+q)$

$\underline{x = a(p+q)}$

$\therefore y = p(a(p+q)) - ap^2$

$y = ap^2 + apq - ap^2$

$\underline{y = apq}$ $\therefore T\,(a(p+q), apq)$

(2)

ii) If $q = p - 1$,

the $T\,(a(p + (p-1)), ap(p-1))$

$\therefore x_T = a(2p-1)$, $y_T = ap(p-1)$

or $\underline{x = 2ap - a}$ (1) $\underline{y = ap^2 - ap}$ (2)

$\therefore 2ap = x + a$

$p = \frac{x+a}{2a}$ (3)

sub (3) into (2) $y = a\left(\frac{x+a}{2a}\right)^2 - a\left(\frac{x+a}{2a}\right)$

$= \cancel{a}\left(\frac{x+a}{2\cancel{a}}\right)\left[\frac{x+a}{2a} - 1\right]$

(3)

$y = \frac{x+a}{2}\left[\frac{x+a-2a}{2a}\right]$

$4ay = (x+a)(x-a)$

$4ay = x^2 - a^2$

$\underline{x^2 = 4ay + a^2} \Rightarrow \underline{\underline{x^2 = 4a\left(y + \frac{a}{4}\right)}}$

GIRRAWEEN HIGH SCHOOL
MATHEMATICS

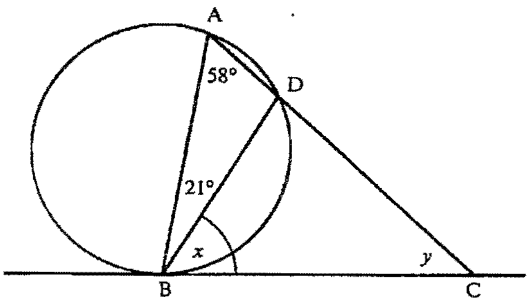
Year 12 Extension 1 Task 1 November 27th 2006

Time Allowed: 90 minutes

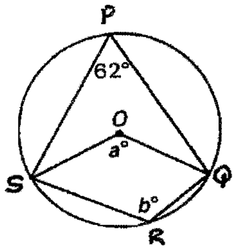
Instructions: Write all your answers on your own paper.
Start each question on a new page.
Show all necessary working.
Marks may be deducted for careless or badly arranged work.

Question1 (12 marks)

a) Find the values of x and y, giving reasons. **4**



b) Find the values of a and b, giving reasons. **4**



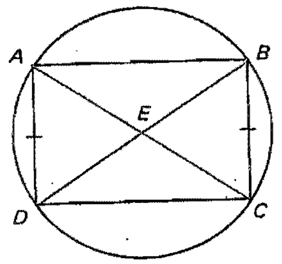
c) Prove by mathematical induction that

$$4+10+18+\ldots+n(n+3)=\frac{1}{3}n(n+1)(n+5)$$ **4**

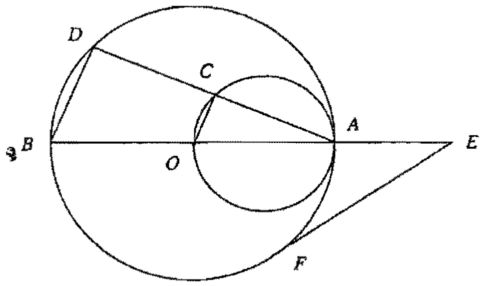
for all positive integers n

Question 2 (16 marks)

a) ABCD is a cyclic quadrilateral. AD = BC. The diagonals meet at E. Prove that AB is parallel to DC. **6**



b) Given that AE = OA, O is the centre of the larger circle and EF is a tangent to the circle,



(i) prove that OC is parallel to BD **3**

(ii) show that EF = $\sqrt{3}$ AE **3**

(iii) prove that ΔOCA is similar to ΔBDA **3**

(iv) if OB = 8cm and BD = 7cm, find the length of OC. **1**

Question 3 (14 marks)

a) Write the expansion of $(1+2x)^5$. **2**

b) Find the 4th term in the expansion of $(x^3 - \frac{2}{x})^8$ **3**

OMIT

c) Find the term independent of x in the expansion of $(3x + \frac{2}{x^2})^9$ **3**

d) If $(1+\sqrt{3})^4 = a + b\sqrt{3}$, find the values of a and b. **3**

e) Find the coefficient of x^6 in the expansion of $(2x-3)^{20}$. **3**
Leave your answer in unexpanded form.

Question 4 (13 marks)

a) For the expansion of $(2+5x)^{12}$,

(i) derive the ratio for $\frac{T_{k+1}}{T_k}$ **3**

OMIT

(ii) hence, find the largest coefficient. **2**

(You may leave your answer in the form ${}^{12}C_k 2^a 5^b$)

b) Prove by mathematical induction that $11^n - 1$ is divisible by 10 for all $n \geq 1$. **4**

c) Prove by mathematical induction that **4**

$$3^n \geq 2n+1 \quad \text{for} \quad n \geq 1$$

SOLUTIONS

Ext 1 HSC Task 1 2006

Question 1 (12 marks)

a) $x = 58°$ (∠ in alternate segment) (2)

$y = 180° - (58+21+58)$

$= 43°$ (∠ sum of Δ) (2)

b) $a = 124°$ (∠ at the centre is twice ∠ at circum. on same arc) (2)

$b = 118°$ (opp. ∠s of cycl. quad suppl.) (2)

c) $4+10+18+\dots+n(n+3) = \frac{1}{3}n(n+1)(n+5)$

Show result is true for $n=1$

LHS $= 1(1+3) = 4$

RHS $= \frac{1}{3}(1)\cdot(2)(6) = \frac{12}{3} = 4$

∴ true for $n=1$

Assume true for $n=k$

ie, $4+10+18+\dots+k(k+3) = \frac{1}{3}k(k+1)(k+5)$

Prove true for $n=k+1$

ie, $4+10+18+\dots k(k+3)+(k+1)(k+4) = \frac{1}{3}(k+1)(k+2)(k+6)$

LHS $= 4+10+18+\dots+k(k+3)+(k+1)(k+4)$

$= \frac{1}{3}k(k+1)(k+5)+(k+1)(k+4)$

$= \frac{1}{3}(k+1)[k(k+5)+3(k+4)]$

$= \frac{1}{3}(k+1)(k^2+8k+12)$

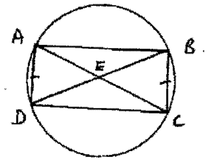
$= \frac{1}{3}(k+1)(k+2)(k+6) =$ RHS

Hence, if the result is true for $n=k$, then it is true for $n=k+1$

Since the result is true for $n=1$, then it is true for $n=1+1=2$, and if true for $n=2$, also true for $n=2+1=3$ and so on for all positive integral values of n (4)

Question 2 (16 marks)

a)



In Δs ADE and BCE

∠ADE = ∠BCE (∠s in same segment equ

∠AED = ∠BEC (vert. opp ∠s)

AD = BC (given)

∴ ΔADE ≡ ΔBCE (AAS)

∴ AE = CE (matching sides of cong. Δs)

DE = BE (matching sides of cong. Δs)

∴ ABCD is a parallelogram (diagonals bisect each other)

∴ AB // DC (opp. sides of parallelogram) (6)

b) OA is the diameter of smaller circle (BE passes through point of contact)

i) ∠OCA = 90° (∠ in semi-circle)

∠ODC = 90° (∠ in semi-circle)

∴ OC // BD (corresp. ∠s equal) (3)

ii) AE = BO + OA + AE

$EF^2 = BE \cdot EA$ (square of tangent equal to product of intercepts)

$= 3AE \cdot EA$

$= 3AE^2$

∴ $EF = \sqrt{3}AE$ (3)

iii) OB = 8cm,) 7cm

In Δs OCA & BDA

∠A is common

∠BDC = ∠OCA (Corresp. ∠s, OC // BD)

∴ ΔOCA ||| ΔBDA (equiangular) (3)

iv) ∴ $\frac{BD}{OD} = \frac{BA}{OA}$ (ratio of matching sides of similar Δs)

$\frac{7}{OC} = \frac{16}{8}$

∴ OC = 3·5 cm (1)

Question 3 (14 marks)

a) $(1+2x)^5$

$= 1+5(2x)+10(2x)^2+10(2x)^3+5(2x)^4+(2x)^5$

$= 1+10x+40x^2+80x^3+80x^4+32x^5$ (2)

b) $T_4 = {}^8C_3\,(x^3)^5\left(\frac{-2}{x}\right)^3$

$= 56\,x^{15}\left(\frac{-8}{x^3}\right)$

$= -448\,x^{12}$ (3)

c) $T_{k+1} = {}^9C_k\,(3x)^{9-k}\left(\frac{2}{x^2}\right)^k$

$= {}^9C_k\,(3^{9-k})(x^{9-k})\frac{2^k}{x^{2k}}$

$= {}^9C_k\,3^{9-k}\,2^k\,x^{9-3k}$

For term independent of x

$9-3k=0$

$k=3$

$T_4 = {}^9C_3\,3^6\cdot 2^3$

$= 489\,888$ (3)

d) $(1+\sqrt{3})^4 = 1+4\sqrt{3}+6(\sqrt{3})^2+4(\sqrt{3})^3+(\sqrt{3})^4$

$= 1+4\sqrt{3}+18+12\sqrt{3}+9$

$= 28+16\sqrt{3}$

∴ $a=28$, $b=16$ (3)

e) $T_{15} = {}^{20}C_{14}\,(2x)^6(-3)^{14}$

∴ coefficient of x^6

$= {}^{20}C_{14}\;2^6\cdot 3^{14}$ (3)

Question 4

a) i) $(2+5x)^{12}$

$T_{k+1} = {}^{12}C_k\,2^{12-k}\,(5x)^k$

$T_k = {}^{12}C_{k-1}\,2^{13-k}\,(5x)^{k-1}$

$\frac{T_{k+1}}{T_k} = \frac{{}^{12}C_k\,2^{12-k}\,(5x)^k}{{}^{12}C_{k-1}\,2^{13-k}\,(5x)^{k-1}}$

$= \frac{12!}{(12-k)!\,k!} \times \frac{(13-k)!\,(k-1)!}{12!} \times \frac{5x}{2}$

$= \frac{13-k}{k} \times \frac{5x}{2}$ (3)

$\frac{T_{k+1}}{T_k} = \frac{5(13-k)}{2k}$

ii) For greatest coefficient

$\frac{5(13-k)}{2k} > 1$

$65-5k > 2k$

$k < 9\cdot 29$

∴ $k=9$

Greatest coeff $= {}^{12}C_9\;2^3\,5^9$ (2)

b) $11^n - 1$ is divisible by 10 for $n \geq 1$

Show true for $n=1$

$11^1 - 1 = 10$ divisible by 10

$\therefore$ true for $n=1$

Assume true for $n=k$

ie. $11^k - 1 = 10p$ where p is an integer

Prove true for $n=k+1$

$11^{k+1} - 1 = 10q$ where q is an integer

$LHS = 11^{k+1} - 1$

$= 11 \cdot 11^k - 1$

$= 11(10p+1) - 1$

$= 110p + 11 - 1$

$= 110p + 10$

$= 10(11p+1) = 10q$ where $q = 11p+1$

$= RHS$

$\therefore$ divisible by 10

Hence, if true for $n=k$, then true for $n=k+1$.

Since true for $n=1$, then it is true for $n=2$ and if true for $n=2$, also true for $n=2+1=3$ and so on for all $n \geq 1$

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c) $3^n \geq 2n+1$ for ≥ 1

Show true for $n=1$

$LHS = 3^1 = 3$

$RHS = 2(1)+1 = 3$

$\therefore$ true for $n=1$

Assume true for $n=k$

ie. $3^k \geq 2k+1$

Prove true for $n=k+1$

ie $3^{k+1} \geq 2(k+1)+1$

$\geq 2k+3$

$LHS = 3^{k+1} = 3 \cdot 3^k$

$\geq 3(2k+1)$

$\geq 6k+3$

$\geq 2k+3$

$\therefore 3^{k+1} \geq 2(k+1)+1$

Hence, if true for $n=k$, then true for $n=k+1$

Since true for $n=1$, then it is true for $n=2$ and if true for $n=2$, also true for $n=2+1=3$ and so on for all $n \geq 1$

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