Question 1 (15 marks)

a) Find, correct to two decimal places:

$$\frac{2.48^2}{\sqrt{3.75-2.93}}$$ **2**

b) Factorise fully:

(i) $x^3 - 2x^2 - 15x$ (ii) $x^3 + 8$ **4**

c) By expressing $\dfrac{4}{2+\sqrt{5}} - \dfrac{1}{9-4\sqrt{5}}$ in its simplest form

show that it is a rational number. **3**

d) Express as a single fraction, leaving the denominator in factorised form:

$$\frac{x+5}{x^2+x} + \frac{x+4}{x^2-x}$$ **3**

e) Solve the following pair of simultaneous equations:

$$7a + 3b = 36$$
$$5a + 2b = 25$$ **3**

Question 2 (12 marks)

a) Solve $|3x-4| + 2 \le 7$ **2**

b) Express $0.\dot{6}\dot{4}$ as a fraction in its simplest form. **2**

c) Find the exact value of:

$$49^{\frac{-1}{2}} \times 27^{\frac{2}{3}}$$ **2**

d) Solve:

$$x^4 - 10x^2 + 21 = 0$$ **2**

e) Find the value of x if: **2**

$$8^{2x-1} = 16^{x+2}$$

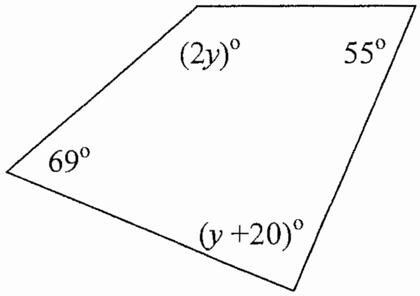
f) Simplify:

$$(\sqrt{2}+1)^2 + (2\sqrt{3})^2$$ **2**

Question 3(12 marks)

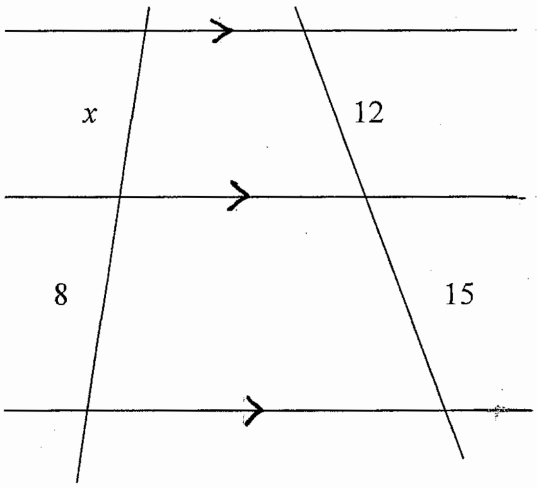
a) Find the size of each interior angle of a regular hexagon. **2**

b) Find the value of y, giving reasons **2**

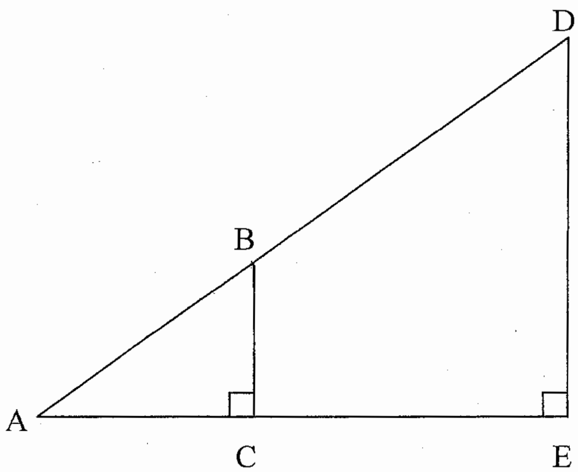


c) Find the value of x, giving reasons. All measurements are in cm.

3



d) **5**



AC = 10cm, CE = 15cm, BC = 8cm

(i) Prove that $\Delta ABC \; ||| \; \Delta ADE$.

(ii) Find the length of DE.

Question 4(21 marks)

a) Find the derivative of:

(i) $5x^3 + 4x - 7 + \frac{3}{x^2}$ **2**

(ii) $(3x^2 - 7)^{10}$ **2**

(iii) $\frac{2x-5}{3x+7}$ **3**

(iv) $\sqrt{4 - x^2}$ **2**

(v) $3x^4(x-4)^5$ **3**

b) Find, from first principles, the gradient of the tangent to the curve $y = 16 - x^2$ at the point where $x = -3$ **3**

c) If $S(t) = 5t^3 - 4t^2 + 5t$, find the value of $S'(-2)$ **2**

d) Find the equation of the normal to the curve $y = 3 + 6x - 2x^2$ at the point (1, 7). **4**

Question 5(22 marks)

a) A(4, 10), B(-3, 1) and C(5, 7) are the vertices of triangle ABC. Plot the points on a number plane. Find

(i) the coordinates of M, the midpoint of BC **1**

(ii) the equation of BC **3**

(iii) the perpendicular distance of A from BC **2**

(iv) the area of ΔABC **2**

b) Prove that the points A(3, -2), B(-1, -7) and C(11, 8) are collinear. **2**

c) Find the exact value of sin 315° **2**

d) Solve: $\sqrt{3}\tan\theta = 1$ for $0° \leq \theta \leq 360°$ **2**

e) Prove that $\tan\theta + \cot\theta = \sec\theta \operatorname{cosec}\theta$ **3**

f) ABCD is a parallelogram in which AB = 4cm, BC = 3cm, $\angle$ ABC = 120°. Find, in exact form

(i) the area of the parallelogram **2**

(ii) the length of the diagonal AC. **3**

Question 6 (20 marks)

a) Draw separate sketches of the following, showing the important features:

(i) $y = |x-3|$ **2**

(ii) $y = 2^x$ **2**

(iii) $y = x^2 - 4$ **2**

b) Indicate clearly on a diagram the region determined by the inequality:

(i) $x + 3y - 3 \le 0$ **3**

(ii) $y < x^2 + 2x$ **3**

c) Draw a clear sketch of the region which satisfies the inequalities

$x^2 + y^2 \le 9$ and $y \ge x + 3$ **4**

d) (i) Sketch the graph of $y = \dfrac{1}{2x-1}$ clearly showing any special features. **2**

(ii) State the domain and range of the function. **2**

Question 7 (17 marks)

a) Find the values of a, b and c if

$2x^2 + 3x - 9 \equiv ax(x-1) + b(x-1) + c$ for all x. **3**

b) For the parabola

$y = 5x^2 + 10x - 2$, find:

(i) the equation of the axis of symmetry **1**

(ii) the coordinates of the vertex **1**

c) The quadratic equation $2x^2 + 4x - 3 = 0$ has roots α and β. Find:

(i) $\alpha + \beta$ (ii) $\alpha\beta$ **2**

(iii) $\alpha^2 + \beta^2$ (iv) $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$ **4**

d) For what values of k is the parabola $y = 2x^2 - kx + 8$ positive definite? **2**

e) (i) Show that there are two values of k for which the equation

$x^2 + (k-6)x + 2k = 0$ has equal roots. **2**

(ii) Determine these roots for each value of k. **2**

Year 11 Yearly 2006
Mathematics Solutions

Question 1 (15 marks)

a) 6·79 ②

b) i) x^3-2x^2-15x

$=x(x^2-2x-15)$

$=x(x-5)(x+3)$ ②

ii) x^3+8

$=(x+2)(x^2-2x+4)$ ②

c) $\frac{4}{2+\sqrt5}-\frac{1}{9-4\sqrt5}$

$=\frac{4(9-4\sqrt5)-(2+\sqrt5)}{(2+\sqrt5)(9-4\sqrt5)}$

$=\frac{34-17\sqrt5}{\sqrt5-2}\times\frac{\sqrt5+2}{\sqrt5+2}$

$=34\sqrt5+68-85-34\sqrt5$

$=-17$ ③

d) $\frac{x+5}{x^2+x}+\frac{x+4}{x^2-x}$

$=\frac{(x+5)(x-1)+(x+4)(x+1)}{x(x+1)(x-1)}$

$=\frac{x^2+4x-5+x^2+5x+4}{x(x+1)(x-1)}$

$=\frac{2x^2+9x-1}{x(x+1)(x-1)}$ ③

e) $7a+3b=36$ — ① ×2

$5a+2b=25$ — ② ×3

$14a+6b=72$ — ③

$15a+6b=75$ — ④

④−③ $a=3$

Sub $a=3$ into ①

$7(3)+3b=36$

$b=5$

$\therefore a=3,\ b=5$ ③

Question 2 (12 marks)

a) $|3x-4|+2\leq 7$

$|3x-4|\leq 5$

$-5\leq 3x-4\leq 5$

$-1\leq 3x\leq 9$

$-\frac{1}{3}\leq x\leq 3$ ②

b) $0\cdot\dot6\dot4$

$x=0\cdot646464\ldots$

$100x=64\cdot646464\ldots$

$99x=64$

$x=\frac{64}{99}$ ②

c) $49^{-1/2}\times 27^{2/3}$

$=\frac{1}{\sqrt{49}}\times(3^3)^{2/3}$

$=\frac{1}{7}\times 9=\frac{9}{7}$ ②

d) $x^4-10x^2+21=0$

Let $u=x^2$

$u^2-10u+21=0$

$(u-7)(u-3)=0$

$u=7,3$

But $u=x^2$

$\therefore x^2=7$ or $x^2=3$

$x=\pm\sqrt7$ or $x=\pm\sqrt3$ ②

e) $8^{2x-1}=16^{x+2}$

$(2^3)^{2x-1}=(2^4)^{x+2}$

$\therefore 6x-3=4x+8$

$x=5\frac{1}{2}$ ②

f) $(\sqrt2+1)^2+(2\sqrt3)^2$

$=2+2\sqrt2+1+12$

$=15+2\sqrt2$ ②

Question 3 (12 marks)

a) ∠ sum of hexagon $=4\times180$

$=720°$

∴ vertex angle $=\frac{720}{6}$

$=120°$ ②

b) $2y+y+20+69+55=360$ (∠ sum of quad $=360°$)

$3y+144=360°$

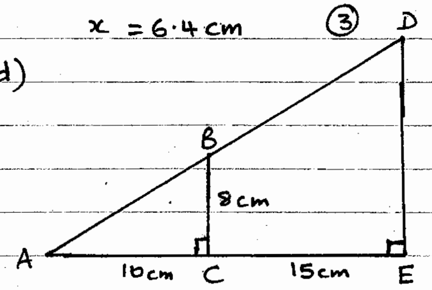
$y=72°$ ②

c) $\frac{x}{8}=\frac{12}{15}$ (intercept theorem)

$15x=96$

$x=6\cdot4$ cm ③

d)



i) ∠BAC = ∠DAE (common ∠)

∠BCA = ∠DEA (given)

∴ ΔABC ||| ΔADE (equiangular) ③

ii) $\frac{DE}{8}=\frac{25}{10}$

DE = 20 cm. ②

iii) $y=\frac{2x-5}{3x+7}$

$y'=\frac{vu'-uv'}{v^2}$

$=\frac{2(3x+7)-3(2x-5)}{(3x+7)^2}$

$=\frac{6x+14-6x+15}{(3x+7)^2}$

$=\frac{29}{(3x+7)^2}$ ③

iv) $y=(4-x^2)^{1/2}$

$y'=\frac{1}{2}(4-x^2)^{-1/2}\cdot -2x$

$=\frac{-x}{\sqrt{4-x^2}}$ ②

v) $y=3x^4(x-4)^5$

$y'=vu'+uv'$

$=12x^3(x-4)^5+15x^4(x-4)^4$

$=3x^3(x-4)^4(4(x-4)+5x)$

$=3x^3(x-4)^4(9x-16)$ ③

Question 4 (21 marks)

a) i) $y=5x^3+4x-7+3x^{-2}$

$\frac{dy}{dx}=15x^2+4-\frac{6}{x^3}$ ②

ii) $y=(3x^2-7)^{10}$

$\frac{dy}{dx}=60x(3x^2-7)^9$ ②

b) $f(x)=16-x^2$

$f'(x)=\lim_{h\to0}\frac{f(x+h)-f(x)}{h}$

$=\lim_{h\to0}\frac{16-x^2-2xh-h^2-16+x^2}{h}$

$=\lim_{h\to0}-2x-h$

$=-2x$

when $x=-3$

$f'(x)=-2(-3)=6$

∴ gradient of tangent at $x=-3$ is 6. ③

4 c) $S(t) = 5t^3 - 4t^2 + 5t$

$S'(t) = 15t^2 - 8t + 5$

$S'(-2) = 15(-2)^2 - 8(-2) + 5$

$= 81$ (2)

d) $y = 3 + 6x - 2x^2$

$\frac{dy}{dx} = 6 - 4x$

At $x = 1$

$m_{tangent} = 2$

$m_{normal} = -\frac{1}{2}$ pt $(1,7)$

Equation of normal:

$y - y_1 = m(x - x_1)$

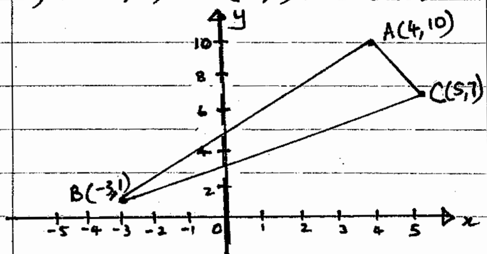
$y - 7 = -\frac{1}{2}(x - 1)$

$2y - 14 = -x + 1$ (4)

$x + 2y - 15 = 0$

Question 5 (22 marks)

a) $A(4,10)$ $B(-3,1)$ $C(5,7)$



i) $M_{BC} = \left(\frac{-3+5}{2}, \frac{1+7}{2}\right)$

$= (1, 4)$ (1)

ii) $m_{BC} = \frac{6}{8} = \frac{3}{4}$ pt $(-3,1)$

Equation of BC

$y - 1 = \frac{3}{4}(x + 3)$

$4y - 4 = 3x + 9$

$3x - 4y + 13 = 0$ (3)

iii) $d_{perp} = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$

$= \frac{|3(4) - 4(10) + 13|}{\sqrt{3^2+4^2}}$

$= 3$ units (2)

iv) $BC = \sqrt{8^2+6^2}$

$= 10$ units

$A_{\Delta ABC} = \frac{1}{2} BC \cdot d_{perp}$

$= \frac{1}{2} \times 10 \times 3$

$= 15$ u^2 (2)

b) $A(3,-2)$ $B(-1,-7)$ $C(11,8)$

$m_{AB} = \frac{-7+2}{-1-3} = \frac{5}{4}$

$m_{BC} = \frac{8+7}{11+1} = \frac{15}{12} = \frac{5}{4}$ (2)

$\therefore$ A, B and C are collinear

c) $\sin 315° = \sin(360° - 45°)$

$= -\sin 45°$

$= -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$ (2)

d) $\sqrt{3}\tan\theta = 1$

$\tan\theta = \frac{1}{\sqrt{3}}$

$\theta = 30°, (180 + 30°)$

$\theta = 30°, 210°$ (2)

e) $\tan\theta + \cot\theta = \sec\theta \operatorname{cosec}\theta$

LHS: $\tan\theta + \cot\theta$

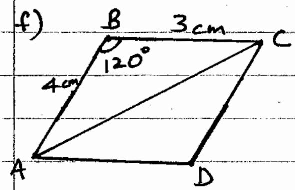
$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta \sin\theta}$

$= \frac{1}{\cos\theta \sin\theta} = \sec\theta \operatorname{cosec}\theta$

$=$ RHS (3)

f)



i) $A = 2\left[\frac{1}{2} \times 4 \times 3 \sin 120°\right]$

$= 6\sqrt{3}$ cm^2 (2)

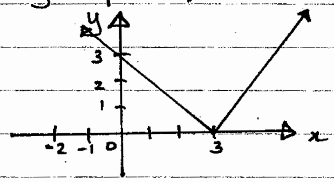
ii) $a^2 = b^2 + c^2 - 2bc\cos A$

$AC^2 = 3^2 + 4^2 - 2(3)(4)\cos 120°$

$AC = \sqrt{37}$ cm (3)

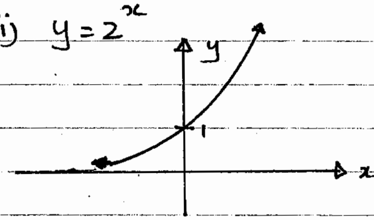
Question 6 (20 marks)

a) i) $y = |x - 3|$



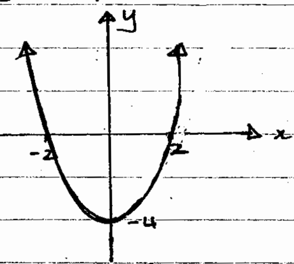
(2)

ii) $y = 2^x$



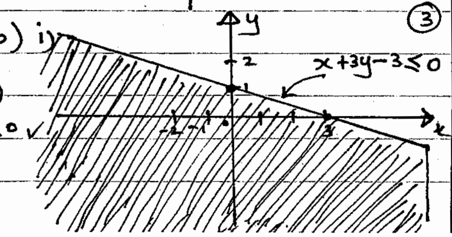
(2)

iii) $y = x^2 - 4$



(2)

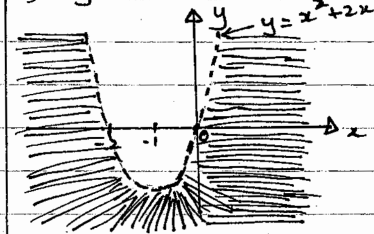
b) i)



Test (0,0)

$0 + 0 - 3 < 0$ ✓ (3)

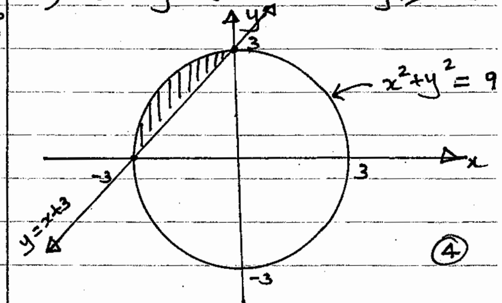
ii) $y < x^2 + 2x$



Test (0,1)

$1 < 0$ (3)

c) $x^2 + y^2 \le 9$ and $y \ge x + 3$

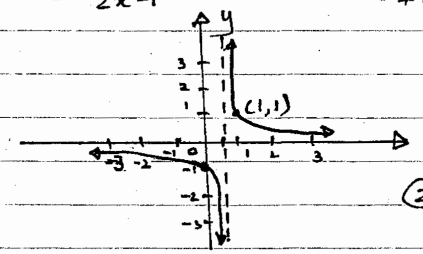


(4)

d) i) $y = \frac{1}{2x-1}$

$2x - 1 \neq 0$

$x \neq \frac{1}{2}$



(2)

ii) Domain : $x \in R$, $x \neq \frac{1}{2}$

Range : $y \in R$, $y \neq 0$ (2)

Question 7 (17 marks)

a) $2x^2+3x-9 \equiv ax(x-1)+b(x-1)+c$

$\equiv ax^2-ax+bx-b+c$

$\equiv ax^2+(b-a)x+c-b$

$\therefore a=2$

$b-a=3$

$b=5$

$c-b=-9$

$c=-4$ (3)

$a=2,\ b=5,\ c=-4$

b) $y=5x^2+10x-2$

i) axis of sym: $x=\frac{-b}{2a}$

$x=-1$ (1)

ii) when $x=-1$

$y=5(-1)^2+10(-1)-2$

$y=-7$

$\therefore$ vertex $=(-1,-7)$ (1)

c) $2x^2+4x-3=0$

i) $\alpha+\beta=\frac{-b}{a}$

$=-2$ (1)

(ii) $\alpha\beta=\frac{c}{a}$

$=\frac{-3}{2}$ (1)

iii) $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=(-2)^2-2\left(\frac{-3}{2}\right)$

$=7$ (2)

iv) $\frac{1}{\alpha}+\frac{1}{\beta}$

$=\frac{\alpha+\beta}{\alpha\beta}$

$=\frac{-2}{-3/2}$

$=\frac{4}{3}$ (2)

d) $y=2x^2-kx+8$

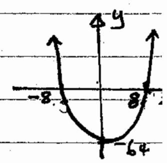
positive definite : $a>0,\ b^2-4ac<0$

$(-k)^2-4(2)(8)<0$

$k^2-64<0$

$(k-8)(k+8)<0$

$\therefore -8<k<8$ (2)



e) i) $x^2+(k-6)x+2k=0$

equal roots : $b^2-4ac=0$

$(k-6)^2-4(1)(2k)=0$

$k^2-12k+36-8k=0$

$k^2-20k+36=0$

$(k-2)(k-18)=0$

$\therefore k=2$ or 18 (2)

$\therefore$ there are two values of k

ii) when $k=2$

$x^2-4x+4=0$

$(x-2)^2=0$

$\therefore x=2$

when $k=18$

$x^2+12x+36=0$

$(x+6)^2=0$

$\therefore x=-6$ (2)

$\therefore$ the roots are 2 and -6

GIRRAWEEN HIGH SCHOOL

MATHEMATICS

YEAR 11 **HSC Task 1, 2006**

Instructions: **Time allowed: 90 minutes**

- Attempt all questions.
- Start each question on a separate page.
- All necessary working must be shown.
- Marks will be deducted for careless or badly arranged work.

Question 1 (16 marks)

(a) For the sequence 5, 8, 11, 14,.... Find 4

(i) the general term.

(ii) the tenth term.

(b) Find the value of m, given that $m+5, 4m+3, 8m-2$ are successive terms of an arithmetic sequence. 2

(c) Evaluate: $\sum_{n=3}^{7}(5n-6)$ 2

(d) Find the sum of all the multiples of 7 between 500 and 1000. 3

(e) The sum of the first four terms of an arithmetic sequence is 34 and the sum of the next four terms is 146. Find the sum of the 9^{th} and 10^{th} terms. 5

Question 2 (13 marks)

(a) Find the equation of the circle with centre at (-3, 4) and radius $4\sqrt{3}$. 3

(b) Find the equation of the locus of a point that moves so that it is equidistant from the points $A(3,2)$ and $B(-1,5)$. 3

(c) Find the centre and radius of the circle $x^2-4x+y^2-10y+4=0$. 3

(d) Find the equation of the locus of a point $P(x,y)$ that moves so that $\angle APB$ is a right angle where $A(4,2)$ and $B(-2,-8)$. 4

Question 3 (16 marks)

(a) Show that $\frac{1}{2}, \frac{1}{3}, \frac{2}{9}, \frac{4}{27}, \ldots\ldots$ is a Geometric sequence. 2

(b) How many terms of the series 2 + 6 + 18 +... are needed to give a sum greater than 500? 3

(c) The fourth, seventh and the last term of a Geometric sequence are 10, 80 and 2560 respectively. Find 5

(i) the first term and the common ratio.

(ii) the number of terms in the Geometric sequence.

(d) Find the limiting sum of the series: $8 + 4\sqrt{2} + 4 + \ldots\ldots\ldots\ldots\ldots$ Write the answer with a rational denominator. 3

(e) An author writes a book, so that on the first day he writes 54 pages, on the second day 36 pages, and so on each succeeding day he writes $\frac{2}{3}$ of the number of pages of the preceding day. Find the total number of pages of the book. 3

Question 4 (15 marks)

(a) Write the equation of the parabola with focus (0,7) and directrix $y = -7$. Sketch the parabola, clearly showing the main features. 4

(b) For the parabola $x^2 = -8y$, find 3

(i) the coordinates of the focus.

(ii) the equation of the directrix.

(iii) the focal length.

(c) For the parabola $x^2 + 6x - 5y - 16 = 0$, find 8

(i) the coordinates of the vertex.

(ii) the coordinates of the focus.

(iii) the equation of the directrix.

(iv) Sketch the parabola, showing the main features.

Question 7 (14 marks)

(a) Find the derivative. 6

(i) $y = (x^2 - 4x)(x^2 + 3)^5$ (ii) $y = \dfrac{2x+3}{x^2 - 5}$

(b) Find the equation of the normal to the curve $y = (3x-2)^3$ at (1, 1). 4

(c) Find the points on the curve $y = x^3 - 2x^2 - x$ at which the tangent lines are parallel to the line $y = 3x - 2$. 4

HSC Mathematics Task 1 - 2006 Solutions

Question 1 (16 marks)

(a) 5, 8, 11, 14 ...

(i) $a = 5$, $d = 3$

$T_n = a + (n-1)d$

$= 5 + (n-1) \times 3$ (2)

$= 5 + 3n - 3$

$= 2 + 3n$

(ii) $T_{10} = 2 + 3 \times 10$ (2)

$= 32$

(b) $4m + 3 - m - 5 = 8m - 2 - 4m - 3$

$3m - 2 = 4m - 5$

$-2 + 5 = 4m - 3m$ (2)

$m = 3$

(c) $\sum_{n=3}^{7} (5n - 6)$

$= (5\times3-6) + (5\times4-6) + (5\times5-6) + (5\times6-6) + (5\times7-6)$

$= 9 + 14 + 19 + 24 + 29$ (2)

$= 95$

(d) 504, ... , 994

$T_n = a + (n-1)d$

$994 = 504 + (n-1)7$

$= 504 + 7n - 7$

$n = 71$

$S_{71} = \frac{71}{2}(504 + 994)$ (3)

$= 53179$

(e) $S_4 = T_1 + T_2 + T_3 + T_4 = 34$

$T_5 + T_6 + T_7 + T_8 = 146$

$S_8 = 34 + 146 = 180$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_4 = \frac{4}{2}[2a + 3d]$

$S_8 = \frac{8}{2}[2a + 7d]$

$34 = 2(2a + 3d)$

$180 = 4(2a + 7d)$

$2a + 3d = 17$ —— (1)

$2a + 7d = 45$ —— (2)

(2) − (1) gives $4d = 28$; $d = 7$

$2a = 17 - 3d$

$= -4$ $\therefore a = -2$

$T_9 + T_{10} = S_{10} - S_8$

$S_{10} = \frac{10}{2}[2\times-2 + 9\times7]$

$= 295$ (5)

$T_9 + T_{10} = 295 - 180$

$= 115$

Question 2 (1 marks)

(a) Centre $(-3, 4)$; radius $= 4\sqrt{3}$

Equation of the circle is (3)

$(x+3)^2 + (y-4)^2 = 48$

(b) $P(x,y)$ $A(3,2)$ $B(-1,5)$

$PA = PB$

$\sqrt{(x-3)^2 + (y-2)^2} = \sqrt{(x+1)^2 + (y-5)^2}$

$(x-3)^2 + (y-2)^2 = (x+1)^2 + (y-5)^2$

$x^2 - 6x + 9 + y^2 - 4y + 4$ (3)

$= x^2 + 2x + 1 + y^2 - 10y + 25$

$8x - 6y + 13 = 0$

(c) $x^2 - 4x + y^2 - 10y + 4 = 0$

$x^2 - 4x + y^2 - 10y = -4$

$x^2 - 4x + 4 + y^2 - 10y + 25 = -4 + 29$

$(x-2)^2 + (y-5)^2 = 25$ (3)

Centre is $(2, 5)$ radius $= 5$

(d) $P(x,y)$ $A = (4,2)$ $B(-2,-8)$

$m_{PA} = \frac{y-2}{x-4}$; $m_{PB} = \frac{y+8}{x+2}$

Given that $PA \perp PB$

$\frac{y-2}{x-4} \times \frac{y+8}{x+2} = -1$

$(y-2)(y+8) = -(x-4)(x+2)$

$(y-2)(y+8) = (4-x)(x+2)$

$y^2 + 8y + 2y - 16 = 4x + 8 - x^2 - 2x$ (4)

$x^2 + y^2 - 2x + 6y - 24 = 0$

Question 3 (16 marks)

(a) $\frac{1}{2}, \frac{1}{3}, \frac{2}{9}, \frac{4}{27}$ ...

$\frac{1}{3} \div \frac{1}{2} = \frac{2}{3}$

$\frac{2}{9} \div \frac{1}{3} = \frac{2}{3}$ (2)

$\frac{T_2}{T_1} = \frac{T_3}{T_2}$

$\therefore$ the given sequence is a Geometric sequence.

(b) $2 + 6 + 18$ ...

$a = 2$ $r = 3$

$S_n = \frac{a(r^n - 1)}{r-1}$

$= \frac{2(3^n - 1)}{3-1} = 3^n - 1$

$3^n - 1 > 500$ (3)

$3^n > 501$

$3^5 = 243$, $3^6 = 729$

$\therefore$ 6 terms are required

(c) $T_4 = 10 \quad T_7 = 80$

last term = 2560

nth term of a GP = ar^{n-1}

$T_4 = ar^3 = 10$

$T_7 = ar^6 = 80$ (2)

$\frac{ar^6}{ar^3} = \frac{80}{10} = 8$

$r^3 = 8 \quad \therefore r = 2$

$a = \frac{10}{r^3} = \frac{10}{8} = \frac{5}{4}$

(ii) Let there be n terms in the GP.

$2560 = \frac{5}{4}(2)^{n-1}$

$512 = \frac{2^{n-1}}{2^2} = 2^{n-3}$ (3)

$2^9 = 2^{n-3}$

$n-3 = 9$

$n = 12$

(d) $8 + 4\sqrt{2} + 4 + \cdots$

$r = \frac{4\sqrt{2}}{8} = \frac{\sqrt{2}}{2}$

$S_\infty = \frac{a}{1-r} = \frac{8}{1-\frac{\sqrt{2}}{2}}$

$= \frac{8}{\frac{2-\sqrt{2}}{2}}$

$= \frac{16}{2-\sqrt{2}}$ (2)

$= \frac{16 \times (2+\sqrt{2})}{(2-\sqrt{2})(2+\sqrt{2})} = \frac{32+16\sqrt{2}}{4-2}$

$= \frac{32+16\sqrt{2}}{2} = \frac{16(2+\sqrt{2})}{2}$ (3)

$= 8(2+\sqrt{2})$

(e) $54,\ 36,\ \frac{2}{3} \times 36 = 24,\ 24 \times \frac{2}{3} = 16 \ldots$

$54, 36, 24, 16 \ldots$

$a = 54 \quad r = \frac{2}{3}$

$S_\infty = \frac{a}{1-r} = \frac{54}{1-\frac{2}{3}}$ (3)

$= \frac{54}{\frac{1}{3}} = 162$

Total number of pages = 162

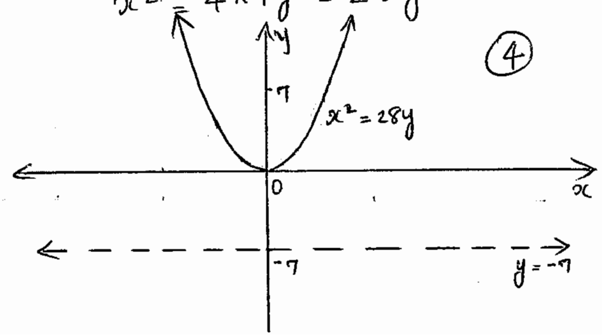
Question 4 (15 marks)

(a) Focus (0,7) directrix $y = -7$

$a = 7$

The equation of the parabola is $x^2 = 4ay$

$x^2 = 4 \times 7y = 28y$ (4)



(b) $x^2 = -8y$

$x^2 = -4ay$

$4a = 8$; $a = 2$

(i) Focus (0, −2) (3)

(ii) directrix : $y = 2$

(iii) Focal length = 2

(c) $x^2 + 6x - 5y - 16 = 0$

$x^2 + 6x = 5y + 16$

$x^2 + 6x + 9 = 5y + 16 + 9$

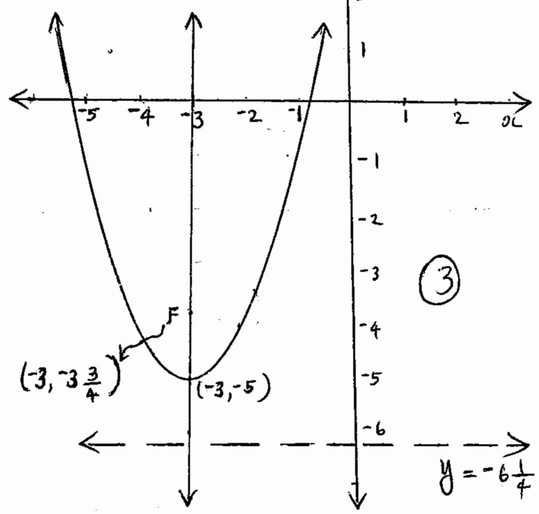
$(x+3)^2 = 5y + 25$

$(x+3)^2 = 5(y+5)$ (5)

(i) Vertex = (−3, −5)

(ii) Focus = $(-3, -3\frac{3}{4})$

(iii) directrix ; $y = -6\frac{1}{4}$



(3)

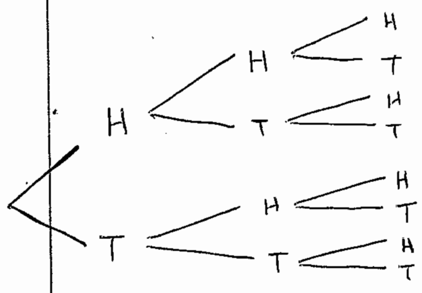
Question 5 (14 marks)

(a) $P(C \text{ or } D) = P(C) + P(D) - P(C \text{ and } D)$

$= 0.5 + 0.4 - 0.3$ (2)

$= 0.6$

(b) 1st IInd IIIrd Sample space

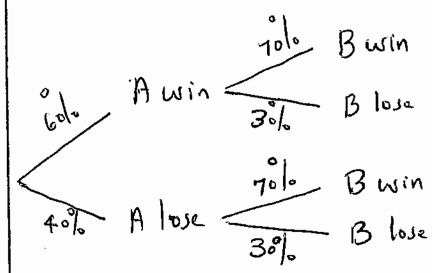


HHH
HHT
HTH
HTT
THH
THT
TTH
TTT

P(2 heads and 1 tail)

$= \frac{3}{8}$ (2)

(c) Seat 1 Seat 2



(2)

(ii) (a) P(both win)

$= \frac{60}{100} \times \frac{70}{100} = \frac{42}{100}$ (2)

(b) P(A win, B lose)

$= \frac{60}{100} \times \frac{30}{100} = \frac{18}{100}$ (2)

(c) $P(A \text{ lose}, B \text{ lose})$

$= \frac{40}{100} \times \frac{30}{100} = \frac{12}{100}$ (2)

(d) $P(\text{at most one wins})$

$= 1 - P(\text{both win})$

$= 1 - \frac{42}{100} = \frac{58}{100}$ (2)

Question 6 (15 marks)

(a) Let A be the event of drawing a ten, B be the event of drawing a picture card, C be the event of drawing a red card

$P(A) = \frac{4}{52}$; $P(B) = \frac{12}{52}$

$P(C) = \frac{26}{52}$

(i) The events A and B are mutually exclusive

$P(A \text{ or } B) = P(A) + P(B)$

$= \frac{4}{52} + \frac{12}{52} = \frac{16}{52}$ (2)

$= \frac{4}{13}$

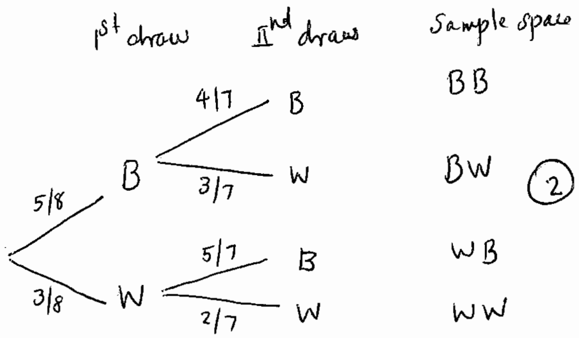
(ii) The events A and C are not mutually exclusive.

$P(A \text{ and } C) = \frac{2}{15}$, as there are two red cards numbered 10. (3)

$P(A \text{ or } C) = P(A) + P(C) - P(A \text{ and } C)$

$= \frac{4}{52} + \frac{26}{52} - \frac{2}{52} = \frac{7}{13}$ (3)

(b) (i)



(2)

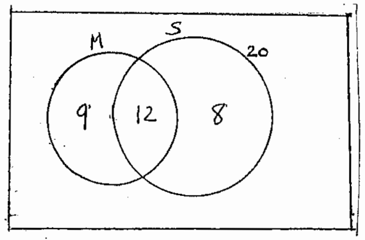
(ii) (a) $P(BB) = \frac{5}{8} \times \frac{4}{7} = \frac{5}{14}$ (2)

(b) $P(BW \text{ or } WB) = P(BW + P(WB)$

$= \left(\frac{5}{8} \times \frac{3}{7}\right) + \left(\frac{3}{8} \times \frac{5}{7}\right)$ (2)

$= \frac{30}{56} = \frac{15}{28}$

(c)



(i) Number of students studying science only $= 17 - 9 = 8$ (2)

Number of students studying both science and Maths $= 20 - 8 = 12$

(ii) P(studen')studies both subjects $) = \frac{12}{29}$ (2)

Question 7 (14 marks)

(a) (i) $y = (x^2 - 4x)(x^2+3)^5$

$y' = (x^2 - 4x) \times 5(x^2+3)^4 \times 2x$

$+ (x^2+3)^5 \times (2x - 4)$ (3)

$= 10x(x^2-4x)(x^2+3)^4 +$

$(2x-4)(x^2+3)^5$

(ii) $y = \frac{2x+3}{x^2-5}$

$y' = \frac{(x^2-5) \times 2 - (2x+3) \times 2x}{(x^2-5)^2}$

$= \frac{2x^2 - 10 - 4x^2 - 6x}{(x^2-5)^2}$ (3)

$= \frac{-2x^2 - 6x - 10}{(x^2-5)^2} = \frac{-2(x^2+3x+5)}{(x^2-5)^2}$

(b) $y = (3x-2)^3$

$y' = 9(3x-2)^2$

$y'|_{x=1} = 9(3-2)^2 = 9$

Gradient of the normal $= \frac{-1}{9}$

Equation of the normal

$y - 1 = -\frac{1}{9}(x-1)$ (4)

$9y - 9 = -(x-1)$

$x + 9y - 10 = 0$

(c) $y = x^3 - 2x^2 - x$

$y' = 3x^2 - 4x - 1$

$3x^2 - 4x - 1 = 3$

$3x^2 - 4x - 4 = 0$ (4)

$(x-2)(3x+2) = 0$

$x = 2$ or $x = \frac{-2}{3}$

when $x = 2$, $y = 8 - 8 - 2 = -2$

when $x = \frac{-2}{3}$, $y = \left(\frac{-2}{3}\right)^3 - 2\left(\frac{-2}{3}\right)^2 - \left(\frac{-2}{3}\right)$

$= -\frac{14}{27}$

Thus the required points are

$(2, -2)$ and $\left(\frac{-2}{3}, \frac{-14}{27}\right)$