

$$\int f'(x)[f(x)]^n dx$$

$$\int f'(x)[f(x)]^n dx = \frac{[f(x)]^{n+1}}{n+1} + c$$

e.g. (i) a) Find $\frac{d}{dx} \left\{ \sqrt{1-x^3} \right\}$

$$\begin{aligned} \frac{d}{dx} \left\{ \sqrt{1-x^3} \right\} &= \frac{1}{2} (1-x^3)^{-\frac{1}{2}} (-3x^2) \\ &= \frac{-3x^2}{2\sqrt{1-x^3}} \end{aligned}$$

b) Hence find; $\int \frac{x^2}{\sqrt{1-x^3}} dx$

$$\begin{aligned} \int \frac{x^2}{\sqrt{1-x^3}} dx &= -\frac{2}{3} \int \frac{-3x^2}{2\sqrt{1-x^3}} dx \\ &= -\frac{2}{3} \sqrt{1-x^3} + c \end{aligned}$$

(ii) $\int x\sqrt{2+x^2} dx$

$$\begin{aligned} &= \frac{1}{2} \int 2x\sqrt{2+x^2} dx \\ &= \frac{1}{2} \cdot \frac{2}{3} (2+x^2)^{\frac{3}{2}} + c \\ &= \frac{1}{3} (2+x^2)\sqrt{2+x^2} + c \end{aligned}$$

OR

$\int x\sqrt{2+x^2} dx$

$$\begin{aligned} &\frac{1}{2} du \\ &= \frac{1}{2} \int u^{\frac{1}{2}} du \\ &= \frac{1}{2} \cdot \frac{2}{3} u^{\frac{3}{2}} + c \\ &= \frac{1}{3} (2+x^2)\sqrt{2+x^2} + c \end{aligned}$$

$$u = 2 + x^2$$

$$\frac{du}{dx} = 2x$$

$$du = 2x dx$$

$$(iii) \int_0^1 \frac{x^2}{(x^3 - 2)^3} dx$$

$$= \frac{1}{3} \int_0^1 \frac{3x^2}{(x^3 - 2)^3} dx$$

$$= \frac{1}{3} \int_0^1 3x^2 (x^3 - 2)^{-3} dx$$

$$= -\frac{1}{6} \left[(x^3 - 2)^{-2} \right]_0^1$$

$$= -\frac{1}{6} \left\{ \frac{1}{(-1)^2} - \frac{1}{(-2)^2} \right\}$$

$$= -\frac{1}{8}$$

OR

$$\int_0^1 \frac{x^2}{(x^3 - 2)^3} dx$$

$$= \frac{1}{3} \int_{-2}^{-1} u^{-3} du$$

$$= -\frac{1}{6} \left[u^{-2} \right]_{-2}^{-1}$$

$$= -\frac{1}{6} \left\{ \frac{1}{(-1)^2} - \frac{1}{(-2)^2} \right\}$$

$$= -\frac{1}{8}$$

$$u = x^3 - 2$$

$$du = 3x^2 dx$$

$$x = 0, u = -2$$

$$x = 1, u = -1$$

Exercise 11H; 1, 3, 5, 7ace etc, 8bdf, 9 11*