



2009
TRIAL
HIGHER SCHOOL CERTIFICATE

Mathematics Extension 1

General Instructions:

- Reading Time - 5 minutes
- Working time - 2 hours
- Write using black or blue pen.
- Board - approved calculators may be used
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown in every question.

Total marks - 84

Attempt Questions 1 - 7

All questions are of equal value

Question 1 (12 marks). Start on a SEPARATE page.

Marks

(a) If $P(x) = x^4 - 3x^3 + ax^2 - 12$ is divisible by $(x-3)$, find the value of a . **2**

(b) i) Find the gradients at the point $P(1,1)$ of the tangents to the **2**
curves $y = x^3$ and $y = 1 - \ln x$.

ii) Hence find the acute angle between these tangents, giving the answer **2**
correct to the nearest degree.

(c) A $(1, - 3)$ and B $(6, 7)$ are two points. Find the coordinates of the **2**
point $P(x,y)$ which divides the interval AB internally in the ratio 2:3

(d) Find $\int \cos^2 4x dx$. **2**

(e) Differentiate $\cos^{-1}(3x)$ with respect to x . **2**

Question 2. (12 marks) start on a **SEPARATE** page.

(a) Use the substitution $u = x - 1$ to find $\int 5x\sqrt{x-1}dx$ **3**

(b) T $(2t, t^2)$ is a point on the parabola $x^2 = 4y$.

i) Show that the tangent to the parabola at T has equation

$$tx - y - t^2 = 0. \quad \mathbf{2}$$

ii) Hence find the values of t such that the tangent to the parabola at T passes through the point P $(1, -2)$. **2**

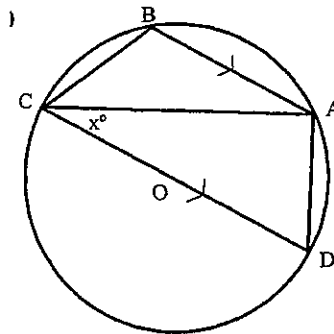
c) i) Express $\cos x - \sqrt{3} \sin x$ in the form $R \cos(x + \alpha)$

where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$ **3**

ii) Hence solve $\cos x - \sqrt{3} \sin x = -2$ for $0 \leq x \leq 2\pi$. **2**

Question 3. (12 marks) .Start on a SEPARATE page.

- (a) The points A, B, C and D lie on the circumference of a circle centred at O.
 CD is a diameter of the circle and AB is parallel to CD. $\angle ACD = x^\circ$.



Find an expression for $\angle ACB$ in terms of x

- 3

- (b) Use the method of mathematical induction to show that the expression $9^n - 8n - 1$ is divisible by 64 for all integers $n \geq 2$.

3

- (c) For the expansion of the expression $(x - \frac{3}{x})^8$, find the term independent of x .

3

- (d) i) Sketch the graph of $y = 2 \sin^{-1} 3x$.

2

- ii) State the domain and range of the function

1

Question 4. (12 marks) .Start on a **SEPARATE** page.

(a) How many groups of 2 men and 2 women can be formed from 6 men and 8 women. 1

(b) Six letter words are formed from the letters of the word CYCLIC.

i) How many different 6-letter words can be formed? 1

ii) How many 6 letter words can be formed, if no "C" s are together.? 2

(c)

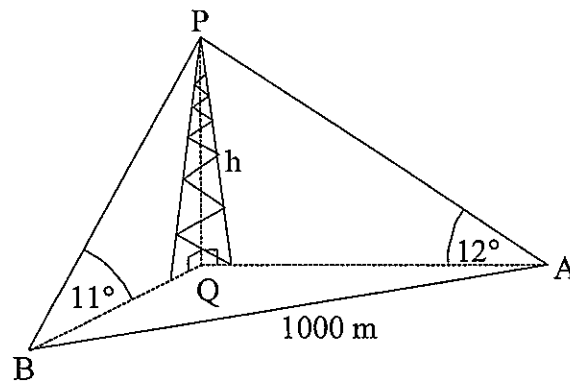


FIGURE NOT TO SCALE

The angle of elevation of a tower PQ of height h metres at a point A due east of it is 12° . From another point B, the bearing of the tower is $051^\circ T$ and the angle of elevation is 11° . The points A and B are 1000 metres apart and on the same level as the base Q of the tower.

- i. Show that $\angle AQB = 141^\circ$. 1
- ii. Consider the triangle APQ and show that $AQ = h \tan 78^\circ$. 1
- iii. Find a similar expression for BQ. 1
- iv. Use the cosine rule in the triangle AQB to calculate h to the nearest metre. 2

Question 4 continues on the next page.

(d) i) Show that : $\frac{2 \tan x}{1 + \tan^2 x} = \sin 2x$ 1

ii) Hence, or otherwise, find $\int_0^{\frac{\pi}{4}} \frac{\tan x}{1 + \tan^2 x} dx$ 2

Question 5. (12 marks) .Start on a SEPARATE page.

(a) The rate at which a body cools in air is proportional to the difference between the temperature, T , of the body and the constant surrounding temperature, S . This can be expressed as

$$\frac{dT}{dt} = k(T - S) \text{ where } t \text{ is time in minutes and } k \text{ is a constant.}$$

i) Show $T = S + Be^{kt}$ where B is a constant, is a solution of the above equation. 1

ii) If a particular body cools from 100° to 80° in 30 minutes, find the temperature of the body after a further 30 minutes, given the surrounding temperature remains constant at 25° . Give your answer to the nearest degree. 4

(b) A particle is moving in a straight line. At time t seconds it has displacement x metres from a fixed point O on the line, velocity $v \text{ ms}^{-1}$ given by $v = 2 - x$ and acceleration $a \text{ ms}^{-2}$.

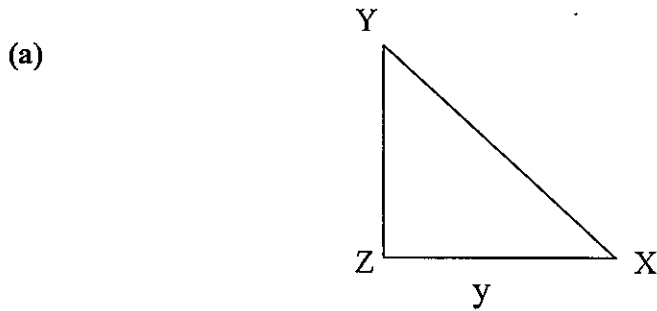
Initially the particle is 4 metres to the left of O .

(i) Find an expression for a in terms of x . 2

(ii) Use integration to show that $x = 2 - 6e^{-t}$ 3

(iii) Find the exact time taken by the particle to travel 4 metres from its starting point. 2

Question 6. (12 marks) .Start on a SEPARATE page.



In $\triangle XYZ$, $ZX = y$ and $\angle YZX = 90^\circ$

i) Show that the area A and perimeter P of the triangle are given by

$$A = \frac{1}{2}y^2 \tan X \text{ and } P = y(1 + \tan X + \sec X) \text{ respectively.} \quad 2$$

ii) If $X = \frac{\pi}{4}$ radians and y is increasing at a constant rate of 0.1 cm s^{-1}

find the rate at which the area of the triangle is increasing at the instant

when $y = 20 \text{ cm}$. 2

iii) If $y = 10 \text{ cm}$ and X is increasing at a constant rate of 0.2 radians per second,

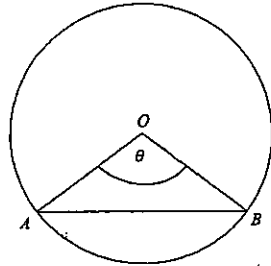
find the rate at which the perimeter of the triangle is increasing

when $X = \frac{\pi}{6}$ radians. 2

Question 6 continues on the next page

Question 6

(b)



AB is a chord of a circle of radius 1 metre that subtends an angle θ at the centre of the circle, where $0 < \theta < \pi$. The perimeter of the minor segment cut off by AB is equal to the diameter of the circle.

(i) Show that $\theta + 2 \sin \frac{1}{2} \theta - 2 = 0$. **2**

(ii) Show that the value of θ lies between 1 and 2 **2**

(iii) Use one application of Newton's method with an initial approximation of $\theta_0 = 1$ to find the next approximation to the value of θ , giving your answer correct to 1 decimal place. **2**

Question 7. (12 marks) .Start on a SEPARATE page.

(a) A ball is projected from a point O on horizontal ground in a room of length R metres with an initial speed of $U \text{ ms}^{-1}$ at an angle of projection of α . There is no air resistance and the acceleration due to gravity is $g \text{ ms}^{-2}$

(i) Assuming after t seconds the ball's horizontal distance x metres, is given by: $x = U t \cos \alpha$, and the vertical component of motion is $\ddot{y} = -g$, show that the vertical displacement y of the ball is given by:

$$y = U t \sin \alpha - \frac{1}{2} g t^2 \quad 2$$

(ii) Hence show that the range R metres for this ball is given by:

$$R = \frac{U^2 \sin 2\alpha}{g} \quad 2$$

(iii) Suppose that the room has a height of 3.5 metres and the angle of projection is fixed for $0 < \alpha < \frac{\pi}{2}$ but the speed of projection U varies.

Prove that: the maximum height will occur when $U^2 = 7g \operatorname{cosec}^2 \alpha$ and the maximum range would be $14 \cot \alpha$. 4

(b)

i) Write down the binomial expansion of $(1-x)^{2n}$ in ascending powers of x 1

ii) Hence show that :

$$\binom{2n}{1} + 3\binom{2n}{3} + \dots + (2n-1)\binom{2n}{2n-1} = 2\binom{2n}{2} + 4\binom{2n}{4} + \dots + 2n\binom{2n}{2n} \quad 3$$

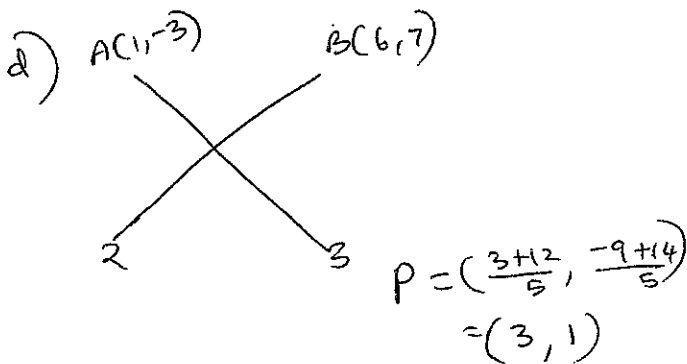
End of paper

Question 1.

a) $P(x) = x^4 - 3x^3 + ax^2 - 12$
 $P(3) = 3^4 - 3 \cdot 3^3 + a \cdot 3^2 - 12 = 0$ (1)
 $\therefore 81 - 81 + 9a - 12 = 0$
 $9a = 12$
 $a = \frac{12}{9} = \frac{4}{3}$ (1)

b) $y = x^3$ $y = 1 - \ln x$
 $y' = 3x^2$ $y' = -\frac{1}{x}$ (2)
 at (1,1), $y' = 3 \cdot 1 = 3$ at (1,1), $y' = -1$
 $\therefore m_1 = 3$ $\therefore m_2 = -1$
 $\tan \theta = \left| \frac{3 - (-1)}{1 - 3} \right| = \left| \frac{4}{-2} \right| = 2$
 $\therefore \theta = \tan^{-1}(2) = 63^\circ$ (2)

c) $\int \cos^2 4x dx$
 $\cos^2 4x = \frac{\cos 8x}{2} + \frac{1}{2}$
 $\therefore \int \cos^2 4x dx = \frac{1}{2} \int (\cos 8x + 1) dx$
 $= \frac{1}{2} \left[\frac{\sin 8x}{8} + x \right] + C$
 $= \frac{\sin 8x}{16} + \frac{x}{2} + C$ (2)



e) $\frac{d}{dx} (\cos^{-1} 3x) = \frac{-1}{\sqrt{1-9x^2}} \times 3$
 $= \frac{-3}{\sqrt{1-9x^2}}$ (2)

Question 2

a) $u = x-1 \therefore \frac{du}{dx} = 1 \Rightarrow du = dx$
 $\int 5x\sqrt{x-1} dx = \int 5(u+1)\sqrt{u} du$
 $= 5 \int (u^{\frac{3}{2}} + u^{\frac{1}{2}}) du$
 $= 5 \left[\frac{2}{5} u^{\frac{5}{2}} + \frac{2}{2} u^{\frac{3}{2}} \right] + C$
 $= 2u^{\frac{5}{2}} + \frac{10}{3} u^{\frac{3}{2}} + C$
 $= 2(x-1)^{\frac{5}{2}} + \frac{10}{3}(x-1)^{\frac{3}{2}} + C$ (3)

b) (i) $4y = x^2$
 $\therefore y = \frac{x^2}{4} \Rightarrow y' = \frac{2x}{4} = \frac{x}{2}$
 at $(2t, t^2)$, $y' = \frac{2t}{2} = t$
 \therefore The equation is
 $y - t^2 = t(x - 2t)$
 $y = tx - t^2 \Rightarrow tx - y - t^2 = 0$ (2)

(ii) Sub. $P(1, -2)$ in the equation
 $tx - y - t^2 = 0$
 $\therefore t + 2 - t^2 = 0 \Rightarrow t^2 - t - 2 = 0$
 $(t-2)(t+1) = 0$
 $\therefore t = 2$ or -1 (2)

e) (i) $\cos x - \sqrt{3} \sin x = R \cos(x + \alpha)$
 $= R \cos x \cos \alpha - R \sin x \sin \alpha$
 $\therefore R \cos \alpha = 1$ and $R \sin \alpha = \sqrt{3}$
 $\therefore \tan \alpha = \sqrt{3} \mid \begin{matrix} R^2 = 4 \\ R = 2 \end{matrix}$
 $\therefore \alpha = \frac{\pi}{3}$
 $\therefore \cos x - \sqrt{3} \sin x = 2 \cos \left(x + \frac{\pi}{3} \right)$ (2)

(ii) $2 \cos \left(x + \frac{\pi}{3} \right) = -2$
 $\therefore \cos \left(x + \frac{\pi}{3} \right) = -1 \Rightarrow x + \frac{\pi}{3} = \cos^{-1}(-1)$
 $(0 \leq x \leq 2\pi)$
 $\therefore x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ (5)

9) $\angle CAD = 90^\circ$ (angle at circumference in semicircle).

$\angle BAC = x^\circ$ (alternate angles are equal, $AB \parallel CD$)

$\therefore \angle BAD = 90^\circ + x^\circ$ (1)

ABCD is a cyclic quadrilateral

\therefore opposite angles are supplementary (2)

$\therefore \angle BCD = 180 - (90 + x) = 90 - x$

$\therefore \angle ACB = \angle BCD - \angle ACD = 90 - x - x = 90 - 2x$ (3)

b) When $n=2$, $T_2 = 9^2 - 16 - 1 = 64$ which is divisible by 64

\therefore True for $n=2$ (1)

Assume that it is true for $n=k, k \geq 2$

$\therefore 9^k - 8k - 1 = 64A$, and $A \in \mathbb{Z}$.

Then $9^{k+1} - 8(k+1) - 1 = 9 \cdot 9^k - 8k - 9 = 9(9^k - 1) - 8k$

$= 9(9^k - 8k - 1) + 64k = 9 \times 64A + 64k = 64(9A + 6k)$ (2)

\therefore true for $n=k+1$

$\therefore 9^n - 8n - 1$ is divisible by 64, $n \geq 2$

c) $(x - \frac{3}{x})^8 = \sum_{r=0}^8 {}^8C_r x^{8-r} (-\frac{3}{x})^r = \sum_{r=0}^8 {}^8C_r (-3)^r x^{8-2r}$ (1)

$8 - 2r = 0 \Rightarrow r = 4$ (2)

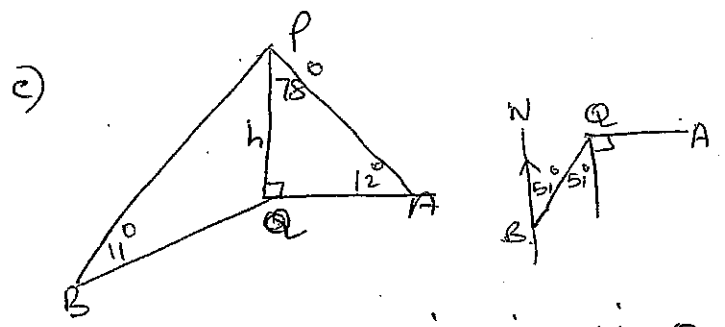
\therefore The term is ${}^8C_4 (-3)^4 = 5670$ (3)

a) ${}^6C_2 \times {}^8C_2 = 420$ (1)

b) (i) Number of words = $\frac{6!}{3!} = 120$ (1)

(ii) $\frac{c}{-} \frac{c}{-} \frac{c}{-} \frac{c}{-} \frac{c}{-}$

Total, if no c's are together = $4 \times 3! = 24$ (2)



(i) $\angle AQB = 90^\circ + 51^\circ = 141^\circ$ (1)

(ii) In $\triangle APQ$, $\angle APQ = 78^\circ$
 $\therefore \frac{AQ}{h} = \tan 78^\circ \Rightarrow AQ = h \tan 78^\circ$ (2)

(iii) In $\triangle PAB$, $\frac{BQ}{h} = \tan 79^\circ$
 $\therefore BQ = h \tan 79^\circ$ (3)

(iv) In $\triangle ABQ$,
 $AB^2 = AQ^2 + BQ^2 - 2AQ \cdot BQ \cos 141^\circ$

$\therefore 1000000 = h^2 \tan^2 78^\circ + h^2 \tan^2 79^\circ - 2h^2 \tan 78^\circ \tan 79^\circ \cos 141^\circ$

$\therefore h^2 = \frac{1000000}{86.2188} \Rightarrow h = 107.69 = 108m$ (2)

(d) (i) $\frac{2 \tan x}{1 + \tan^2 x} = \frac{2 \frac{\sin x}{\cos x}}{\sec^2 x}$

$$= \frac{2 \sin x}{\cos x} \times \cos^2 x$$

$$= 2 \sin x \cdot \cos x$$

$$= \sin 2x \quad (1)$$

(i) $\int_0^{\frac{\pi}{4}} \frac{\tan x}{1 + \tan^2 x} dx = \frac{1}{2} \int_0^{\frac{\pi}{4}} \sin 2x dx$

$$= -\frac{1}{4} [\cos 2x]_0^{\frac{\pi}{4}}$$

$$= -\frac{1}{4} [\cos \frac{\pi}{2} - \cos 0]$$

$$= -\frac{1}{4} [0 - 1] = \frac{1}{4} \quad (2)$$

Questions

a) (i) $\frac{dT}{dt} = k \cdot B e^{kt}$

$$= k(T - S) \quad (1)$$

$T = 100$
 $t = 30$

(ii) $100 = 25 + B e^{k \times 30} \Rightarrow B = 75$ (1)

$T = 80$
 $t = 30$

$$80 = 25 + 75 e^{30k}$$

$$\frac{55}{75} = e^{30k}$$

$$\frac{11}{15} = e^{30k}$$

$$\log_e \left(\frac{11}{15} \right) = 30k$$

$$\therefore k = \frac{1}{30} \log_e \left(\frac{11}{15} \right) \quad (2)$$

$t = 60, T = ?$

$$T = 25 + 75 e^{60k}$$

$$= 65.33^\circ$$

$$= 65^\circ \quad (1)$$

b) (i) $v = 2 - x$

$$a = \frac{dv}{dx} = (2 - x)(-1)$$

$$= x - 2 \quad (2)$$

(i) $v = \frac{dx}{dt} = 2 - x$

$$\therefore \frac{dt}{dx} = \frac{1}{2 - x}$$

$$dt = \frac{dx}{2 - x}$$

$$\int dt = \int \frac{dx}{2 - x} \quad (1)$$

$$t = -\ln(2 - x) + c$$

$t = 0, x = -4$

$$\therefore 0 = -\ln 6 + c$$

$$\Rightarrow c = \ln 6$$

$$\therefore t = -\ln(2 - x) + \ln 6$$

$$\therefore t = \ln \frac{6}{2 - x} \quad (1)$$

$$\therefore e^t = \frac{6}{2 - x}$$

$$2e^t - xe^t = 6$$

$$\therefore xe^t = 2e^t - 6$$

$$x = 2 - 6e^{-t} \quad (1)$$

(iii) When $t = 0, x = -4$

After the particle has travelled 4m from its starting position,

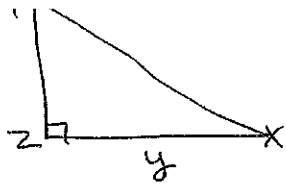
$$x = 0 \Rightarrow 0 = 2 - 6e^{-t}$$

$$\therefore 6e^{-t} = 2$$

$$e^{-t} = \frac{2}{6} = \frac{1}{3}$$

$$\therefore t = \ln 3 \quad (2)$$

a) (i)



$$\tan x = \frac{zy}{y} \Rightarrow zy = y \tan x$$

$$\begin{aligned} \therefore \text{Area} &= \frac{1}{2} \times y \tan x \times y \\ &= \frac{1}{2} y^2 \tan x \quad \text{--- (1)} \end{aligned}$$

$$\frac{y}{xy} = \cos x \Rightarrow xy = y \sec x$$

$$\begin{aligned} \therefore \text{Perimeter } P &= y + y \tan x + y \sec x \\ &= y [1 + \tan x + \sec x] \quad \text{--- (1)} \end{aligned}$$

(ii) $\frac{dy}{dt} = 0.1 \text{ cm s}^{-1}$

$$\frac{dA}{dt} = \frac{dA}{dy} \times \frac{dy}{dt}$$

$$A = \frac{1}{2} y \tan^2 x$$

$$\begin{aligned} \therefore \frac{dA}{dy} &= y \tan x \\ &= y \tan \frac{\pi}{4} \text{ when } x = \frac{\pi}{4} \\ &= y \quad \text{--- (1)} \end{aligned}$$

$$\begin{aligned} \therefore \frac{dA}{dt} &= y \times 0.1 = 20 \times 0.1 \\ &= 2 \text{ cm}^2/\text{sec.} \quad \text{--- (1)} \end{aligned}$$

(iii) $\frac{dx}{dt} = 0.2 \text{ radians/sec.}$

$$\frac{dP}{dt} = \frac{dP}{dx} \times \frac{dx}{dt}$$

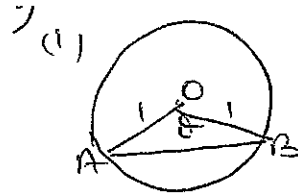
$$P = y(1 + \tan x + \sec^2 x)$$

$$\begin{aligned} \therefore \frac{dP}{dx} &= y(\sec^2 x + \sec x \tan x) \quad \text{--- (1)} \\ &= 10 \left(\sec^2 \frac{\pi}{6} + \sec \frac{\pi}{6} \tan \frac{\pi}{6} \right) \end{aligned}$$

when $y = 10, x = \frac{\pi}{6}$

$$= 10 \left[\frac{4}{3} + \frac{2}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}} \right] = 10 \left[\frac{4}{3} + \frac{2}{3} \right]$$

$$\therefore \frac{dP}{dt} = 20 \times 0.2 = 4 \text{ cm/sec.} \quad \text{--- (1)}$$



Using cosine rule,

$$\begin{aligned} AB^2 &= 1^2 + 1^2 - 2 \cos \theta \\ &= 2(1 - \cos \theta) \\ &= 4 \cdot \sin^2 \frac{1}{2} \theta \\ \therefore AB &= 2 \cdot \sin \frac{1}{2} \theta \end{aligned}$$

$$\text{Arc } AB = r\theta = \theta \quad (r=1)$$

$$\begin{aligned} \therefore \text{Perimeter} &= \text{diameter} \Rightarrow \\ \theta + 2 \sin \frac{1}{2} \theta &= 2 \end{aligned}$$

$$\therefore \theta + 2 \sin \frac{1}{2} \theta - 2 = 0. \quad \text{--- (2)}$$

(ii) $f(\theta) = \theta + 2 \sin \frac{1}{2} \theta - 2$

$$f(1) = 1 + 2 \sin \frac{1}{2} - 2 \approx -0.04$$

$$f(2) = 2 + 2 \sin 1 - 2 \approx 1.68 > 0$$

Since $f(\theta)$ is continuous,
 $f(\theta) = 0$ for some $1 < \theta < 2$ --- (2)

(iii) $f(\theta) = \theta + 2 \sin \frac{\theta}{2} - 2$

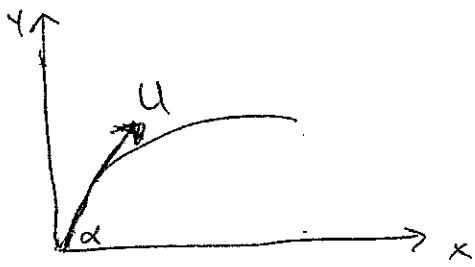
$$f'(\theta) = 1 + \cos \frac{\theta}{2}$$

$$\begin{aligned} \therefore \theta_1 &= \theta_0 - \frac{f(\theta_0)}{f'(\theta_0)} \\ &= 1 - \frac{-1 + 2 \sin \frac{1}{2}}{1 + \cos \frac{1}{2}} \end{aligned}$$

$$\approx 1.0 \text{ (to one decimal place)}$$

(2)

(i) $t=0, x=0$ and $y=0$ and $\begin{matrix} u \\ \alpha \\ \dot{y} = u \sin \alpha \\ \dot{x} = u \cos \alpha \end{matrix}$



$$\ddot{y} = -g$$

$$\dot{y} = \int g dt = -gt + C$$

when $t=0, \dot{y} = u \sin \alpha \Rightarrow C = u \sin \alpha$

$$\therefore \dot{y} = -gt + u \sin \alpha$$

$$\therefore y = \int (-gt + u \sin \alpha) dt = u t \sin \alpha - \frac{gt^2}{2} + D$$

$t=0, y=0 \Rightarrow D=0$ (2)

$$\therefore y = u t \sin \alpha - \frac{gt^2}{2} \dots (i)$$

(ii) For the range, $y=0$

$$\therefore t \left(u \sin \alpha - \frac{gt}{2} \right) = 0$$

$$\therefore t=0 \text{ or } t = \frac{2u \sin \alpha}{g}$$

$$\begin{aligned} \therefore \text{Range} = x &= u t \cos \alpha \\ &= u \cdot \frac{2u \sin \alpha}{g} \cdot \cos \alpha \\ &= \frac{u^2 \sin 2\alpha}{g} \end{aligned} \quad (2)$$

(iii) At maximum height, $\dot{y}=0$

$$\therefore u \sin \alpha - gt = 0$$

$$\therefore t = \frac{u \sin \alpha}{g}$$

sub $t = \frac{u \sin \alpha}{g}$ in (i)

$$\text{maximum height} = 3.5 = \frac{u \cdot u \sin \alpha}{g} - \frac{g}{2} \cdot \frac{u^2 \sin^2 \alpha}{g^2}$$

$$3.5 = \frac{u^2 \sin^2 \alpha}{2g} \Rightarrow u^2 = \frac{7g}{\sin^2 \alpha} = 7g \operatorname{cosec}^2 \alpha \quad (2)$$

(iv) Maximum range $R = \frac{u^2 \sin 2\alpha}{g}$

$$= \frac{7g}{\sin^2 \alpha} \times \frac{2 \sin \alpha \cos \alpha}{g}$$

$$= \frac{14 \cos \alpha}{\sin \alpha}$$

$$= 14 \cot \alpha \quad (2)$$

b) (i)

$$(1-x)^{2n} = {}^{2n}C_0 - {}^{2n}C_1 x + {}^{2n}C_2 x^2 - {}^{2n}C_3 x^3 + \dots + {}^{2n}C_{2n-1} (-x)^{2n-1} + {}^{2n}C_{2n} x^{2n} \quad (1)$$

(ii) By differentiating both sides w.r.t.

$$-2n(1-x)^{2n-1} = -2n {}^{2n}C_1 + 2 \cdot {}^{2n}C_2 x - 3 \cdot {}^{2n}C_3 x^2 + \dots$$

$$+ 2n \cdot {}^{2n}C_{2n-1} x^{2n-1} \quad (1)$$

Sub. $x=1$ both sides,

$$0 = -2n {}^{2n}C_1 + 2 \cdot {}^{2n}C_2 - 3 \cdot {}^{2n}C_3 + \dots$$

$$- (2n-1) {}^{2n}C_{2n-1} + 2n \cdot {}^{2n}C_{2n}$$

$$\therefore 2n {}^{2n}C_1 + 3 \cdot {}^{2n}C_3 + \dots + (2n-1) {}^{2n}C_{2n-1}$$

$$= 2 \cdot {}^{2n}C_2 + 4 \cdot {}^{2n}C_4 + \dots + 2n \cdot {}^{2n}C_{2n} \quad (1)$$