NORTH SYDNEY GIRLS HIGH SCHOOL

2009
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading Time – 5 minutes
- Working Time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total Marks – 84

Attempt Questions 1–7
All questions are of equal value

At the end of the examination, place your solution booklets in order and put this question paper on top.
Submit one bundle.
The bundle will be separated before marking commences so that anonymity will be maintained.

Student Number: ____________________________ **Teacher:** ___________________________

Student Name: ______________________________

QUESTION	MARK
1	/12
2	/12
3	/12
4	/12
5	/12
6	/12
7	/12
TOTAL	/84

Total Marks – 84
Attempt Questions 1–7
All questions are of equal value

Begin each question in a SEPARATE writing booklet. Extra writing booklets are available.

Question 1 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Solve $\dfrac{x-2}{x+3} > -2$. **3**

(b) Find the exact value of $\cos 2x$ if $\sin x = \sqrt{3} - 1$. **2**

(c) The graphs of $y = \dfrac{1}{x}$ and $y = x^3$ intersect at $x = 1$. Find the size of the acute angle between these two curves at $x = 1$. **3**

(d) Use the table of standard integrals to find $\int \sec 3x \tan 3x \, dx$. **1**

(e) Using the substitution $u = 1 + t$, find the exact value of $\int_0^1 \dfrac{t}{\sqrt{1+t}} \, dt$. **3**

Question 2 (12 marks) Use a SEPARATE writing booklet.

(a) Given α, β, γ are the roots of the equation $2x^3 + 3x^2 - 2x - 4 = 0$, evaluate

(i) $\alpha + \beta + \gamma$ **1**

(ii) $\alpha^2 + \beta^2 + \gamma^2$ **1**

(b) The point $P\left(2ap, ap^2\right)$ lies on the parabola $x^2 = 4ay$

(i) Show that the equation of the normal to the curve of the parabola at the point P is $x + py = 2ap + ap^3$. **2**

(ii) Find the co-ordinates of the point Q where the normal at P meets the y-axis. **1**

(iii) The point R divides PQ externally in the ratio 3:1. Show that the co-ordinates of the point R are $\left(-ap, 3a + ap^2\right)$. **2**

(iii) Find the cartesian equation of the locus of R. **1**

(c) (i) Find the co-efficient of x^2 in the expansion of $\left(ax + \dfrac{b}{x^2}\right)^{11}$. **2**

(ii) If this co-efficient of x^2 is equal to the co-efficient of x^{-1}, show that **2**

$$a - 2b = 0.$$

Question 3 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find $\dfrac{d}{dx}\sin(\log_e x)$. **2**

(b) (i) Show that $(k+1)(k+2)$ is even for any positive integer. **1**

(ii) Prove by mathematical induction that $n(n+1)(n+2)$ is divisible by 6 for all positive integers n. **3**

(c) There are three boxes *A*, *B* and *C* into which different cards are placed without regard to the order of cards within any box.

(i) Find the total number of ways 10 cards may be distributed so that there are 5 cards in *A*, 3 in *B* and 2 in *C*. **1**

(ii) Four cards are distributed at random between *A*, *B* and *C*. Find the probability that each box will contain at least one card. **3**

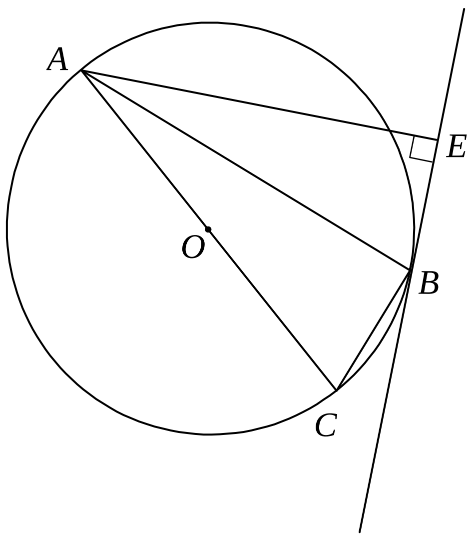
(iii) Now only three cards are to be distributed between the boxes *A*, *B* and *C*. Assuming the probability of a card being placed in box *A* is $\dfrac{3}{10}$ and in box *B* is $\dfrac{2}{5}$, find the probability that each of the three boxes will contain a card. **2**

Question 4 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) (i) Find the value of the constant a for which the polynomial $P(x) = x^4 + 2x^3 - x^2 - 8x - a$ is divisible by $Q(x) = x^2 - 4$. **1**

(ii) Hence or otherwise find all real zeros of the polynomial $P(x)$ with that particular value of a. **2**

(b) Two points A and B lie on a circle and AC is the diameter.
AE is perpendicular to the tangent at B.



(i) Copy or trace the diagram onto your paper.

(ii) Prove that AB bisects $\angle CAE$. **3**

(c) A function is defined by the equation $y = \dfrac{e^x}{x^2+1}$.

(i) Describe the behaviour of the function for large positive and negative values of x. **1**

(ii) Show that $\dfrac{dy}{dx} = \dfrac{e^x(x-1)^2}{(x^2+1)^2}$. **2**

(iii) Determine the co-ordinates of the stationary point(s), without considering the second derivative. **1**

(iv) Sketch the curve showing all important features. **2**

Question 5 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Fiona is one of 7 executive members of a travel company. Each year one member is selected at random to win a round the world ticket.

(i) What is the probability that in the first 4 years Fiona will win at least one ticket? **1**

(ii) Show that in the first 25 years Fiona has a greater chance of winning exactly 3 tickets than exactly 2 tickets. **2**

(iii) How many years should Fiona work for the travel company to be 90% certain of winning at least one round the world ticket? **2**

(b) (i) For $x > 0$, show that **2**

$$\log_2\left(\frac{1}{x}\right)+\log_2\left(\frac{1}{x^2}\right)+\log_2\left(\frac{1}{x^3}\right)+\ldots\ldots$$

is an arithmetic series, and hence find the common difference.

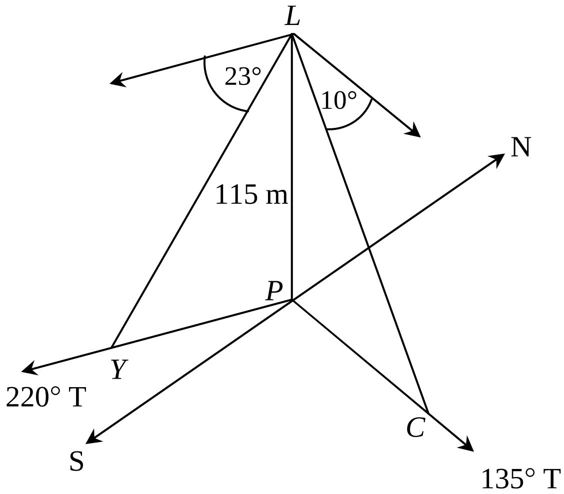
(ii) If the sum of the first ten terms is 440, find the value of x. **2**

(c) Use the identity $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$ to solve the equation $\sin 3\theta = 2\sin\theta$ for $0 \le \theta \le 2\pi$. **3**

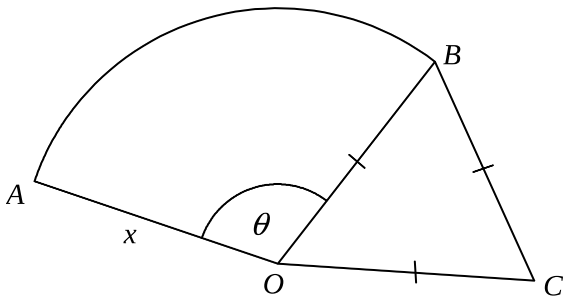
Question 6 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) From the top of a lighthouse, L, 115 metres above sea level, a container ship, C, is seen on a bearing of 135°T, at an angle of depression of 10°. **3**
A yacht, Y is also sighted on a bearing of 220°T, at an angle of depression of 23°.
This is illustrated in the diagram below.

Calculate the distance between the two vessels, correct to the nearest metre.



(b) The diagram shows a sector OAB of a circle, centre O and radius x metres. Arc AB subtends an angle of θ radians at O. An equilateral triangle BCO adjoins the sector.



(i) Write an expression in terms of θ and x for

(α) the perimeter of $OABC$. **1**

(β) the area of $OABC$. **2**

(ii) Given that the perimeter has the value $\left(12-2\sqrt{3}\right)$ metres, show that the area A is given by **2**

$$A=\frac{\left(6-\sqrt{3}\right)^2\left(2\theta+\sqrt{3}\right)}{\left(\theta+3\right)^2}$$

(iii) For which value of θ is the area a maximum? Justify your answer. **4**

Question 7 (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Given $\dfrac{dy}{dx} = \cos^2 x$ and $y = 1$ when $x = 0$ find y in terms of x. **2**

(b) The surface area A of a solid of revolution generated by rotating the part of a curve between $x = a$ and $x = b$ about the x axis is given by

$$\int_a^b 2\pi y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx.$$

(i) The curve $y = 2\sqrt{x}$ is rotated about the x axis. Show that the surface area generated over the interval $0 \le x \le 8$ is given by **2**

$$4\pi\int_0^8 (x+1)^{\frac{1}{2}}\,dx.$$

(ii) Hence find the required surface area. **2**

(c) (i) Show that, for all positive integers n, **2**

$$1+(1+x)+(1+x)^2+\,.....\,+(1+x)^n = \frac{(1+x)^{n+1}-1}{x}.$$

(ii) Hence, or otherwise, show that for all integers $n \ge 2$, **2**

$$\binom{2}{2}+\binom{3}{2}+\binom{4}{2}+\,.....\,+\binom{n}{2}=\binom{n+1}{3}.$$

(iii) The polynomial $1-x+x^2-x^3+.....+x^{16}-x^{17}+x^{18}$ may be written in the form **2**

$$b_0+b_1w+b_2w^2+\,.....\,+b_{17}w^{17}+b_{18}w^{18}$$

where $w = x+1$ and $b_0, b_1, ...,b_{18}$ are real numbers.
Using the results of parts (i) and (ii) above, or otherwise, find the value of b_2.

End of paper

Solutions

Extension 1
NSGH Trial 2009

Q1 a) $\frac{x-2}{x+3}(x+3)^2 > -2(x+3)^2$

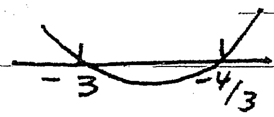
$0 > -2(x+3)^2 - (x-2)(x+3)$

$2(x+3)^2 + (x-2)(x+3) > 0$

$(x+3)[2(x+3) + x-2] > 0$

$(3x+4)(x+3) > 0$

$x < -3$ or $x > -\frac{4}{3}$



b) $\cos 2x = 1 - 2\sin^2 x$

$= 1 - 2(\sqrt{3}-1)^2$

$= 1 - 2(3 - 2\sqrt{3} + 1)$

$= -7 + 4\sqrt{3}$

c) $y = \frac{1}{x}$

$\frac{dy}{dx} = -\frac{1}{x^2}$

At $x=1$, $\frac{dy}{dx} = -1$

$y = x^3$

$\frac{dy}{dx} = 3x^2$

At $x=1 = 3$

$\tan\theta = \left|\frac{3 - -1}{1+3(-1)}\right|$

$\theta = \tan^{-1} 2$

$= 63° \, 26'$

d) $\int \sec 3x \tan 3x \, dx = \frac{1}{3}\sec 3x + C$

e) $\int_0^1 \frac{t}{\sqrt{1+t}} \, dt$

$u = 1+t$

$du = dt$

$= \int_1^2 \frac{u-1}{u^{1/2}} \, du$

When $t=1$, $u=2$

When $t=0$, $u=1$

$= \int_1^2 u^{1/2} - u^{-1/2} \, du$

$= \left[\frac{2u^{3/2}}{3} - 2u^{1/2}\right]_1^2 = \frac{4-2\sqrt{2}}{3}$

Q2 a)(i) $\alpha+\beta+\gamma = \frac{-3}{2}$

(ii) $\alpha^2+\beta^2+\gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\alpha\gamma+\beta\gamma)$

$= \left(\frac{-3}{2}\right)^2 - 2\left(\frac{-2}{2}\right)$

$= \frac{17}{4}$ or $4\frac{1}{4}$

b) $y = \frac{1}{4a}x^2$

$\frac{dy}{dx} = \frac{1}{2a}x$

At P, $\frac{dy}{dx} = \frac{2ap}{2a} = p$

Equation of normal at P:

$y - ap^2 = -\frac{1}{p}(x-2ap)$

$py - ap^3 = -x + 2ap$

$x + py = 2ap + ap^3$

(ii) Normal meets y axis when $x=0$

$0 + py = 2ap + ap^3$

$y = 2a + ap^2$

Q has co-ord $(0, 2a+ap^2)$

(iii) $P(2ap, ap^2)$ $\quad Q(0, +2a+ap^2)$

$-3 : 1$

$x = \frac{1\times 2ap - 3\times 0}{-3+1}$ and $y = \frac{1\times ap^2 - 3(2a+ap^2)}{-3+1}$

$= \frac{-2ap^2-6a}{-2}$

R has co-ord $(-ap, 3a+ap^2)$

(iii) (iv) If $x = -ap$ then $p = \frac{-x}{a}$

$y = 3a + a\left(-\frac{x}{a}\right)^2$

$= 3a + \frac{x^2}{a}$

$\therefore$ Locus of R is $y = \frac{x^2}{a} + 3a$

2 c) (i) $\left(ax+\frac{b}{x^2}\right)^{11} = {}^{11}C_0 (ax)^{11}\left(\frac{b}{x^2}\right)^0 + \ldots\ldots$

General term $= {}^{11}C_k (ax)^{11-k}(bx^{-2})^k$

x^2 term : $x^{11-k}x^{-2k} = x^2$

$$x^{11-3k} = x^2$$

$$11-3k = 2$$

$$k = 3$$

$\therefore x^2$ term is ${}^{11}C_3\, a^8 x^8 (b)^3 . x^{-6}$

Coefficient of x^2 is ${}^{11}C_3\, a^8 b^3$ or ${}^{11}C_3\, a^8 b^3$

(ii) x^{-1} term : $x^{11-k}. x^{-2k} = x^{-1}$

$$11-3k = -1$$

$$k = 4$$

Coefficient of x^{-1} is ${}^{11}C_4\, a^7 (b)^4$ or ${}^{11}C_4\, a^7 b^4$

Now ${}^{11}C_3\, a^8 b^3 = {}^{11}C_4\, a^7 b^4$ Note negativ

$$\frac{11!}{8!\,3!} a^8 b^3 = \frac{11!}{7!\,4!} a^7 b^4$$

$$165\, a = 330\, b$$

$$a = 2b$$

$a - 2b = 0$ as required

Q3 a) $\frac{d}{dx} \sin(\log_e x) = \cos(\log_e x) \times \frac{1}{x}$

$= \frac{1}{x} \cos(\log_e x)$

b) Let k be even, then $k = 2m$, $m \in \mathbb{Z}^+$

$(k+1)(k+2) = (2m+1)(2m+2)$
$= 2(2m+1)(m+1)$
which is even

Let k be odd, then $k = 2m+1$, $m = \mathbb{Z}^+$

$(k+1)(k+2) = (2m+2)(2m+3)$
$= 2(m+1)(2m+3)$
which is even

$\therefore (k+1)(k+2)$ is even for any positive integer

(ii) Test for $n=1$
$LHS = 1(1+1)(1+2)$
$= 6$, which is divisible by 6
$\therefore$ Result is true for $n=1$

Assume true for some $n=k$, ie $k(k+1)(k+2) = 6M$,

We wish to show true for $n=k+1$, ie $(k+1)(k+2)(k+3) = 6N$,

$LHS = (k+1)(k+2)(k+3)$

$= k(k+1)(k+2) + 3(k+1)(k+2)$
$= 6M + 3 \times 2P$ by assumption and by pa
$= 6(M+P)$, which is a multiple of 6 $P \in \mathbb{Z}$
$\therefore$ Statement is true for $n=k+1$
$\therefore$ By mathematical induction statement is true for all

3 c) (i) $^{10}C_5 \times {}^{5}C_3 \times {}^{2}C_2 = 2520$ ways

(ii) Put two cards in A: $^{4}C_2$ ways

One card in B, one in C: $^{2}C_1 \times {}^{1}C_1$

If there is at least one card in each box, then 1 box has 2 cards

$= {}^{4}C_2 \times {}^{2}C_1 \times {}^{1}C_1$ ways

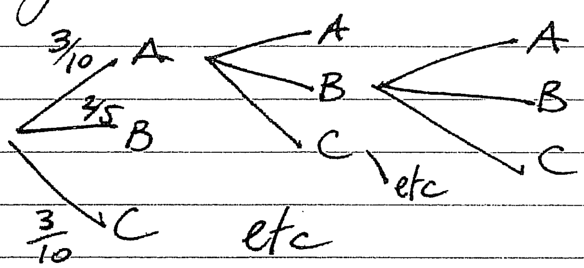
$= 12$ ways

But 2 cards could go in A or B or C 3 ways

$\therefore$ Total $= 3 \times {}^{4}C_2 \times {}^{2}C_1 \times {}^{1}C_1$

$= 36$ ways

(iii) Only 3 cards, and each box must contain one card



Suitable outcomes:

ABC or ACB

BAC or BCA

CAB or CBA

all equally likely

$\therefore 6 \times \left(\frac{3}{10}\right)\left(\frac{2}{5}\right)\left(\frac{3}{10}\right)$

$= 0.216$ or $\frac{27}{125}$

Total ways, without restriction $= 3^4$

$\therefore$ Probability(at least one card in each box) $= \frac{36}{3^4}$

$= \frac{4}{9}$

Q4 $P(x) = x^4 + 2x^3 - x^2 - 8x - a$

$P(-2) = (-2)^4 + 2(-2)^3 - (-2)^2 - 8 \quad -a$

$= 16 - 16 - 4 + 16 - a$

If $P(x)$ is divisible by x^2-4 then $P(-2) = 0$

$\therefore a = 12$

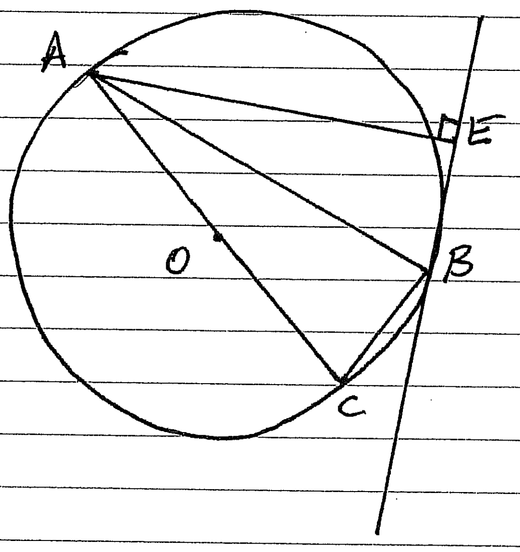
(ii)

$$\begin{array}{r}
x^2 + 2x + 3 \\
x^2 - 4 \overline{\smash{)}\, x^4 + 2x^3 - x^2 - 8x - 12} \\
\underline{x^4 \qquad - 4x^2} \\
2x^3 + 3x^2 - 8x \\
\underline{2x^3 \qquad - 8x} \\
3x^2 \qquad - 12 \\
\underline{3x^2 \qquad - 12} \\
0
\end{array}$$

$\therefore P(x) = (x^2-4)(x^2+2x+3)$

Since x^2+2x+3 has no real zeroes $(\Delta < 0)$ then the only real zeros are -2 and 2

b)



(ii) In ΔAEB and ΔABC

$\angle ABC = 90°$ (angle in a se

$\therefore \angle ABC = \angle AEB$

$\angle ABE = \angle ACB$ (angle between c
and tangent)

$\therefore \Delta AEB \,|||\, \Delta ABC$ (equiangular

$\therefore \angle EAB = \angle BAC$ (matching angles of similar

$\therefore$ AB bisects $\angle CAE$

4 c) $y = \dfrac{e^x}{x^2+1}$

For $x \to \infty$, e^x dominates (x^2+1), so $y \to \infty$

For $x \to -\infty$, $e^x \to 0$ and $\dfrac{1}{x^2+1} \to 0$ so $y \to 0$

(ii) $\dfrac{dy}{dx} = \dfrac{(x^2+1)(e^x) - e^x(2x)}{(x^2+1)^2}$ by quotient rule

$= \dfrac{e^x(x^2+1-2x)}{(x^2+1)^2}$

$= \dfrac{e^x(x-1)^2}{(x^2+1)^2}$ as required

(iii) Since $e^x \neq 0$, then $\dfrac{dy}{dx} = 0$ only when $x = 1$, $y = \dfrac{e^1}{1^2+1}$

Stat point is $\left(1, \dfrac{e}{2}\right)$

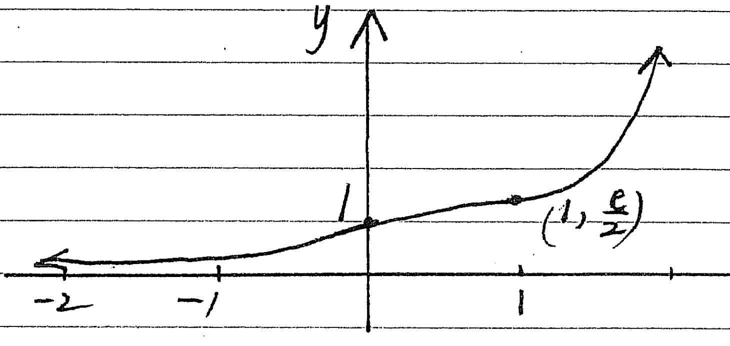
(iv) Nature of stat point :

x	0.9	1	1.1
$\frac{dy}{dx}$	>0	0	>0

There is a horizontal point of inflexion at $\left(1, \dfrac{e}{2}\right)$

When $x = 0$, $y = 1$

Also $y > 0$ for all x



$P(\text{win}) = \frac{1}{7}$

5 a) $P(\text{at least one win}) = 1 - P(\text{no wins in 4 years})$

$= 1 - {}^4C_0 \left(\frac{6}{7}\right)^4 \left(\frac{1}{7}\right)^0$

$= \frac{1105}{2401}$ or 0.46 (2 d.p)

(ii) $P(\text{winning exactly 3 tickets}) = {}^{25}C_3 \left(\frac{6}{7}\right)^{22} \left(\frac{1}{7}\right)^3$

$= 0.2257$

$P(\text{winning exactly 2 tickets}) = {}^{25}C_2 \left(\frac{6}{7}\right)^{23} \left(\frac{1}{7}\right)^2$

$= 0.17666$ (5 d.p)

$\therefore P(X=3) > P(X=2)$ where X chance of winning the no of tickets exactly

(iii) $P(\text{at least one win in } n \text{ years}) = 1 - P(\text{no wins in } n \text{ years})$

$1 - {}^nC_0 \left(\frac{6}{7}\right)^n \left(\frac{1}{7}\right)^0 > 0.9$

$0.1 > \left(\frac{6}{7}\right)^n$ $\qquad \left(\frac{6}{7}\right)^n < 0.1$

$\log 0.1 > \log\left(\frac{6}{7}\right)^n$

$\frac{\log 0.1}{\log 6/7} < n$ since $\log\left(\frac{6}{7}\right) < 0$

$14.9 < n$

$\therefore$ Fiona should work for 15 years to be 90% cer

5 b) If its an AP, then $T_3 - T_2 = T_2 - T_1$

$T_3 - T_2 = \log_2 \frac{1}{x^3} - \log_2\left(\frac{1}{x^2}\right)$

$= -3\log_2 x - (-\log_2 x)$

$= -\log_2 x$

$T_2 - T_1 = \log_2\left(\frac{1}{x^2}\right) - \log_2\left(\frac{1}{x}\right)$

$= -2\log_2 x + \log_2 x$

$= -\log_2 x$

$\therefore$ Its an AP, with common difference $(-\log_2 x)$

(ii) $S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

$440 = \frac{10}{2}\left(2\log_2\left(\frac{1}{x}\right) + 9(-\log_2 x)\right)$

$= 5\left(-2\log_2 x - 9\log_2 x\right)$

$= -55\log_2 x$

$\therefore \log_2 x = \frac{440}{-55}$

$= -8$

$\therefore x = 2^{-8}$ or $\frac{1}{2^8}$ or $\frac{1}{256}$

c) $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$ and $\sin 3\theta = 2\sin\theta$

$\therefore 2\sin\theta = 3\sin\theta - 4\sin^3\theta$

$0 = \sin\theta - 4\sin^3\theta$

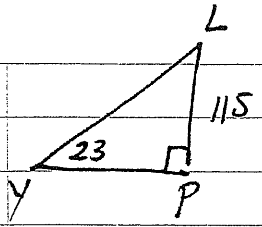
$0 = \sin\theta(1 - 4\sin^2\theta)$

$\therefore \sin\theta = 0$ or $\sin\theta = \pm\frac{1}{2}$

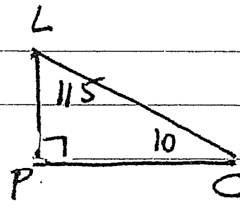
$\theta = 0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}, 2\pi$

Q6

a)



$$PY = \frac{115}{\tan 23^\circ}$$
$$= 115 \tan 67^\circ$$



$$PC = \frac{115}{\tan}$$
$$= 115 \text{ ta}$$

By the cosine rule

$$\text{Distance}^2 = PY^2 + PC^2 - 2PY.PC \times \cos 85^\circ$$

Distance = 684 nearest metre (using exact values PY and PC)

b)(α) $\text{Perimeter} = x\theta + 3x$

(β) $\text{Area} = \frac{1}{2}x^2\theta + \frac{1}{2}x^2 \sin\frac{\pi}{3}$ from area of $\Delta = \frac{1}{2}ab\sin C$

$$= \frac{1}{2}x^2\left(\theta + \frac{\sqrt{3}}{2}\right)$$

(ii) Let perimeter $= 12 - 2\sqrt{3}$ then

$$x(\theta+3) = 12 - 2\sqrt{3}$$
$$x = \frac{12-2\sqrt{3}}{\theta+3}$$

$$\therefore \text{Area} = \frac{1}{2}\left(\frac{12-2\sqrt{3}}{\theta+3}\right)^2\left(\theta + \frac{\sqrt{3}}{2}\right) \text{ from } (\beta)$$

$$= \frac{1}{2} \times \frac{4(6-\sqrt{3})^2}{(\theta+3)^2}\left(\frac{2\theta+\sqrt{3}}{2}\right)$$

$$= \frac{(6-\sqrt{3})^2(2\theta+\sqrt{3})}{(\theta+3)^2} \text{ as required}$$

$$\frac{dA}{d\theta} = (6-\sqrt{3})^2\left[\frac{(\theta+3)^2(2) - (2\theta+\sqrt{3})(2)(\theta+3)}{(\theta+3)^4}\right]$$

$$= (6-\sqrt{3})^2\left[\frac{(\theta+3)(2\theta+6-4\theta-2\sqrt{3})}{(\theta+3)^4}\right]$$

$$= (6-\sqrt{3})^2 \cdot \frac{6-2\sqrt{3}-2\theta}{(\theta+3)^3}$$

$\frac{dA}{d\theta} = 0$ if area is to be a max $\therefore 6-2\sqrt{3}-2\theta = 0$

$2\theta = 6-2\sqrt{\ }$

6 (cont)

θ	1	$3-\sqrt{3}$	3
$\frac{dA}{d\theta}$	>0	0	<0

When $\theta = 3-\sqrt{3}$, a maximum value of A occurs

Q7 $y = \int \cos^2 x \, dx$

$= \frac{1}{2}\int \cos 2x + 1 \, dx$

$= \frac{1}{2}\left(\frac{1}{2}\sin 2x + x\right) + c$

$= \frac{1}{4}\sin 2x + \frac{x}{2} + c$

When $x=0$, $y=1 \Rightarrow c=1$

$\therefore y = \frac{1}{4}\sin 2x - \frac{x}{2} + 1$

b) $y = 2\sqrt{x}$

$\frac{dy}{dx} = \frac{1}{2}.2.x^{-1/2}$

$= \frac{1}{\sqrt{x}}$

$\int_0^8 2\pi 2\sqrt{x}\sqrt{1+\left(\frac{1}{\sqrt{x}}\right)^2}\, dx$

$= \int_0^8 4\pi \sqrt{x}\sqrt{\frac{x+1}{x}}\, dx$

$= \int_0^8 4\pi\sqrt{x+1}\, dx$ as required

(ii) Surface area $= 4\pi . 2\left[\frac{(x+1)^{3/2}}{3}\right]_0^8$

$= \frac{8\pi}{3}(27-1)$

$= \frac{208\pi}{3}$ units^2

c) (i) Geometric series, $r = (1+x)$ and $a = 1$, $S_n = \frac{a(r^n - 1)}{r-1}$

There are $(n+1)$ terms

$\therefore$ Sum to $(n+1)$ terms $= \dfrac{1 \times ((1+x)^{n+1} - 1)}{1+x-1}$

$$= \frac{(1+x)^{n+1} - 1}{x}$$

(ii) On LHS, terms in x^2 are, in general, ${}^nC_2\, x^2$ from $(1+x$

$0 + 0 + \binom{2}{2} x^2 + \binom{3}{2} x^2 + \binom{4}{2} x^2 + \ldots \binom{n}{2} x^2$

on RHS term in x^2 is found from x^3 term $\div x$

ie $\dfrac{\binom{n+1}{3} x^3}{x}$

$\therefore \binom{2}{2} + \binom{3}{2} + \binom{4}{2} + \ldots + \binom{n}{2} = \binom{n+1}{3}$

(iii) If $w = x+1$, then $x = w-1$

$\therefore 1 - x + x^2 - x^3 + \ldots + x^{16} - x^{17} + x^{18}$

$= 1 - (w-1) + (w-1)^2 - (w-1)^3 + \ldots - (w-1)^{17} + (w-1)^{18}$

$= b_0 + b_1 w + b_2 w^2 + \ldots$

The coefficient of w^2 is b_2

In $(-1)^k (w-1)^k$ the coefficient of w^2 is $(-1)^k \binom{k}{2} w^2 (-1)^{k-2}$

$= \binom{k}{2} w^2$

From part (ii), $\sum_{k=2}^{19} \binom{k}{2} w^2 = \binom{19}{3} w^2$

$\therefore b_2 = \binom{19}{3}$