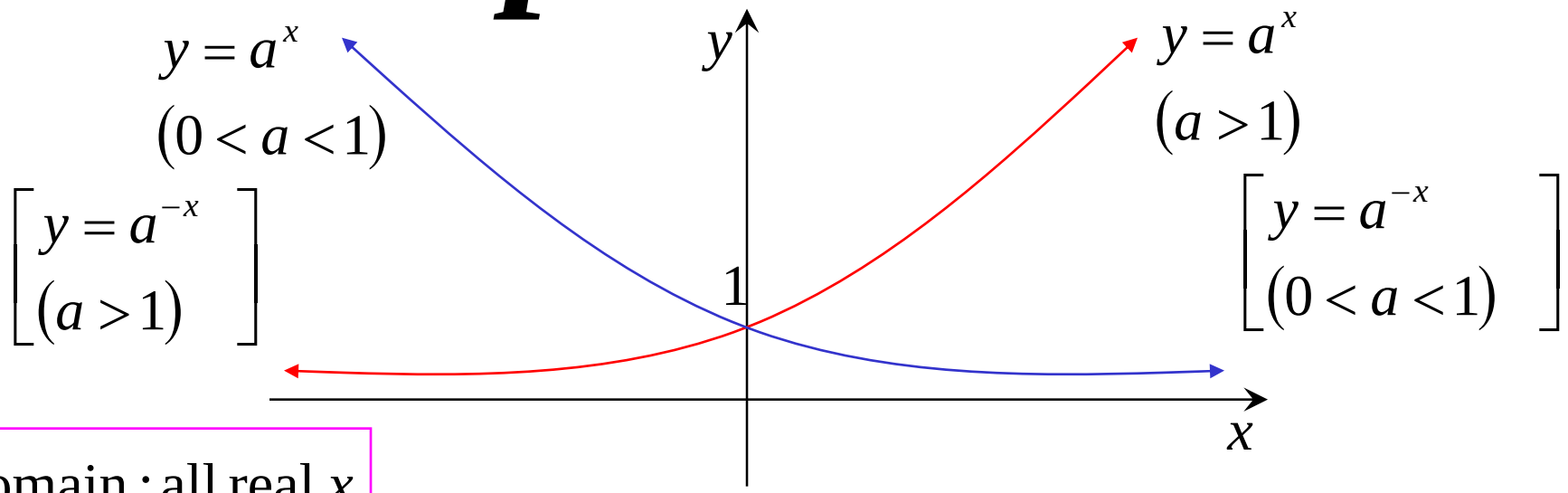
Exponentials



domain : all real x

range : $y > 0$

$y = e^x$

e is an irrational number, it is defined as; $\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$

$e \approx 2.718281828\ldots$

e.g. $e^3 = 20.086$ (to 3 dp)

Differentiating Exponentials

$y = e^{f(x)}$ $\frac{dy}{dx} = f'(x)e^{f(x)}$	$y = a^{f(x)}$ $\frac{dy}{dx} = f'(x)(\log a)a^{f(x)}$

e.g. $(i)\ y = e^{x}$ $\quad (ii)\ y = e^{5x}$ $\quad (iii)\ y = e^{4x+3}$ $\quad (iv)\ y = e^{x^2+3x+2}$

$\frac{dy}{dx} = e^{x}$ $\quad \frac{dy}{dx} = 5e^{5x}$ $\quad \frac{dy}{dx} = 4e^{4x+3}$ $\quad \frac{dy}{dx} = (2x+3)e^{x^2+3x+2}$

$(v)\ y = 3x^2 e^{4x}$

$$\frac{dy}{dx} = (3x^2)(4e^{4x}) + (e^{4x})(6x)$$
$$= 12x^2 e^{4x} + 6xe^{4x}$$
$$= 6xe^{4x}(2x+1)$$

$(vi)\ y = (e^{3x}+2)^7$

$$\frac{dy}{dx} = 7(e^{3x}+2)^6(3e^{3x})$$
$$= 21e^{3x}(e^{3x}+2)^6$$

$(vii)\ y = \frac{e^x}{e^x+3}$

$$\frac{dy}{dx} = \frac{(e^x+3)(e^x) - (e^x)(e^x)}{(e^x+3)^2}$$
$$= \frac{e^{2x} + 3e^x - e^{2x}}{(e^x+3)^2}$$
$$= \frac{3e^x}{(e^x+3)^2}$$

$(viii)$ Find the tangent to $y = e^{2x} + 1$ at the point $(1, e^2+1)$

$$y = e^{2x} + 1$$
$$\frac{dy}{dx} = 2e^{2x}$$

when $x = 1$, $\frac{dy}{dx} = 2e^2$

$$y - (e^2+1) = 2e^2(x-1)$$
$$y - e^2 - 1 = 2e^2 x - 2e^2$$
$$2e^2 x - y - e^2 + 1 = 0$$

$(ix)\quad y = 4^{x^2}$

$$\frac{dy}{dx} = 2x(\log 4)4^{x^2}$$

$(x)\quad y = \log x$

$$x = e^y$$

$$\frac{dx}{dy} = e^y$$

$$\frac{dy}{dx} = \frac{1}{e^y}$$

$$= \frac{1}{x}$$

Exercise 13A; 1 to 4 ace etc, 6 to 8 ace, 10 to 12 ac

Exercise 13B; 4, 7, 8 to 22 evens (not 18)