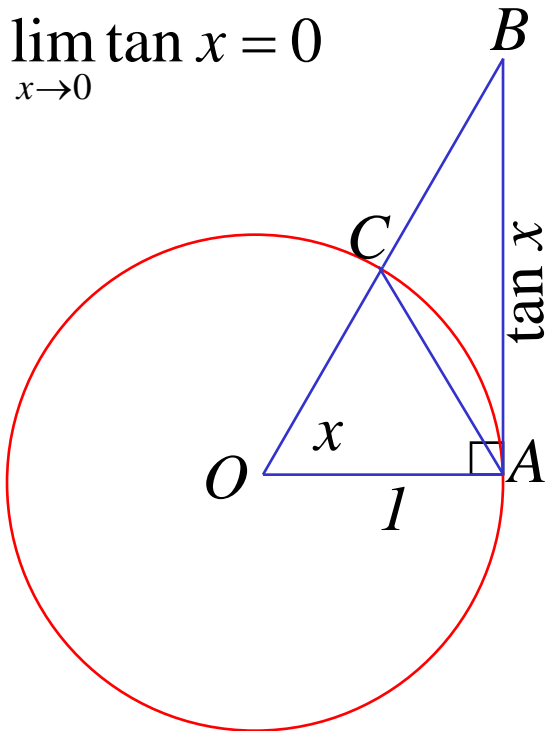


Differentiating Trig

$$\lim_{x \rightarrow 0} \sin x = 0$$

$$\lim_{x \rightarrow 0} \cos x = 1$$

$$\lim_{x \rightarrow 0} \tan x = 0$$



Area $\triangle AOC <$ Area Sector $OAC <$ Area $\triangle AOB$

$$\frac{1}{2}(1)(1)\sin x < \frac{1}{2}(1)^2 x < \frac{1}{2}(1)\tan x$$

$$\sin x < x < \tan x$$

$$\frac{\sin x}{\sin x} < \frac{x}{\sin x} < \frac{\tan x}{\sin x}$$

$$1 < \frac{x}{\sin x} < \frac{1}{\cos x}$$

as $x \rightarrow 0$

$$1 < \frac{x}{\sin x} < 1$$

$$\lim_{x \rightarrow 0} \frac{x}{\sin x} = 1$$

$$\text{e.g. (i) } \lim_{x \rightarrow 0} \frac{5x}{\sin 5x} = \underline{1}$$

$$\text{(ii) } \lim_{x \rightarrow 0} \frac{x}{\sin 3x} = \lim_{x \rightarrow 0} \frac{1}{3} \times \frac{3x}{\sin 3x}$$

$$\underline{y = \sin x}$$

$$= \underline{\frac{1}{3}}$$

$$\frac{dy}{dx} = \lim_{x \rightarrow 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{x \rightarrow 0} \frac{\sin x \cosh + \cos x \sinh - \sin x}{h}$$

$$= \lim_{x \rightarrow 0} \cos x \left(\frac{\sinh}{h} \right) + \sin x \left(\frac{\cosh - 1}{h} \right)$$

$$= \lim_{x \rightarrow 0} \cos x \left(\frac{\sinh}{h} \right) + \sin x \left(\frac{2 \cos^2 \frac{h}{2} - 2}{h} \right) \quad [\cos 2\theta = 2 \cos^2 \theta - 1]$$

$$= \lim_{x \rightarrow 0} \cos x \left(\frac{\sinh}{h} \right) + \sin x \left(\frac{2 \sin^2 \frac{h}{2}}{h} \right)$$

$$\begin{aligned}
&= \lim_{x \rightarrow 0} \cos x \left(\frac{\sinh}{h} \right) + \sin x \left(\frac{\sin^2 \frac{h}{2}}{\frac{h}{2}} \right) \\
&= \lim_{x \rightarrow 0} \cos x \left(\frac{\sinh}{h} \right) + \sin x \left(\frac{\sin \frac{h}{2}}{\frac{h}{2}} \times \sin \frac{h}{2} \right) \\
&= (\cos x)(1) + (\sin x)(0) \\
&= \underline{\cos x}
\end{aligned}$$

$$\underline{y = \cos x}$$

$$y = \sin \left(\frac{\pi}{2} - x \right)$$

$$\frac{dy}{dx} = -\cos \left(\frac{\pi}{2} - x \right)$$

$$\underline{= -\sin x}$$

$$\begin{aligned}
y &= \sin f(x) \\
\frac{dy}{dx} &= f'(x) \cos f(x)
\end{aligned}$$

$$\begin{aligned}
y &= \cos f(x) \\
\frac{dy}{dx} &= -f'(x) \sin f(x)
\end{aligned}$$

$$y = \tan x$$

$$y = \frac{\sin x}{\cos x}$$

$$\frac{dy}{dx} = \frac{(\cos x)(\cos x) - (\sin x)(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x}$$

$$= \sec^2 x$$

$$y = \tan f(x)$$

$$\frac{dy}{dx} = f'(x) \sec^2 f(x)$$

e.g. (i) $y = \sin x^3$

$$\frac{dy}{dx} = 3x^2 \cos x^3$$

(ii) $y = \tan \frac{1}{x}$

$$\frac{dy}{dx} = -\frac{1}{x^2} \sec^2 \frac{1}{x}$$

(iii) $y = \log \cos x$

$$\frac{dy}{dx} = \frac{-\sin x}{\cos x}$$

$$= -\tan x$$

(iv) $y = \tan^5 x$

$$\frac{dy}{dx} = 5 \tan^4 x \sec^2 x$$

(v) $y = \cos e^x$

$$\frac{dy}{dx} = -e^x \sin e^x$$

Exercise 14F; 2, 4, 5

Exercise 14G; 2ace etc, 3ace etc, 5ace etc, 6, 7ab(i), 8, 12, 13c, 16a*

Exercise 14H; 2 ac, 4bd, 6, 8, 14, 17, 20