Integrating Trig

$$\int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b)+c$$

$$\int \sin(ax+b)\,dx = -\frac{1}{a}\cos(ax+b)+c$$

$$\int \sec^2(ax+b)\,dx = \frac{1}{a}\tan(ax+b)+c$$

e.g. $(i)\ \int \sin 3x\,dx = -\frac{1}{3}\cos 3x + c$

$(ii)\ \int \cos(1-5x)\,dx = -\frac{1}{5}\sin(1-5x)+c$

$(iii)\ \int \sec^2\left(\frac{x}{2}\right)dx = 2\tan\left(\frac{x}{2}\right)+c$

$$(iv)\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin 2x\,dx = \left[-\frac{1}{2}\cos 2x\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$$= -\frac{1}{2}\left(\cos\pi - \cos\frac{\pi}{3}\right)$$

$$= -\frac{1}{2}\left(-1 - \frac{1}{2}\right)$$

$$= \frac{3}{4}$$

(v) Find the volume of the solid formed when $y = \sqrt{\sin \pi x}$ between $x = 0$ and $x = 1$ is rotated around the x axis.

$$V = \pi \int y^2 dx$$

$$= \pi \int_0^1 \sin \pi x\,dx$$

$$= \pi \left[-\frac{1}{\pi}\cos \pi x\right]_0^1$$

$$= -(\cos\pi - \cos 0)$$

$$= -(-1-1)$$

$$= 2 \text{ units}^3$$

$$(vi)\int x\sec^2 x^2 dx = \frac{1}{2}\int 2x\sec^2 x^2 dx$$

$$= \frac{1}{2}\tan x^2 + c$$

$$(vii)\int \sin^2 x\,dx$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$= 1 - 2\sin^2\theta \Rightarrow \sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$$

$$= 2\cos^2\theta - 1 \Rightarrow \cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$$

$$= \frac{1}{2}\int (1-\cos 2x)\,dx$$

$$= \frac{1}{2}\left(x - \frac{1}{2}\sin 2x\right) + c$$

$$= \frac{x}{2} - \frac{1}{4}\sin 2x + c$$

$$(vii)\int \tan x\,dx \quad = \int \frac{\sin x}{\cos x}\,dx$$

$$= -\int \frac{-\sin x}{\cos x}\,dx$$

$$= -\log\cos x + c$$

$$= \log(\cos x)^{-1} + c$$

$$= \log\sec x + c$$

$$(ix)\int_0^{\frac{\pi}{2}} \cos x \sin^7 x dx$$

$$u = \sin x$$
$$du = \cos x dx$$
$$x = 0, u = 0$$
$$x = \frac{\pi}{2}, u = 1$$

$$= \int_0^1 u^7 du$$

$$= \left[\frac{1}{8}u^8\right]_0^1$$

$$= \frac{1}{8}\left(1^8 - 0\right)$$

$$= \frac{1}{8}$$

Exercise 14I; 2ace etc, 3ace etc, 4, 6, 8a, 9ac, 10a, 12ace, 13b(i), 14df, 15ace

Exercise 14J; 2b, 3bfh, 4a, 5ac, 7, 9, 10, 13, 14, 21, 26