Finance Formulae

Simple Interest

$$\boxed{I = PRT}$$

$I =$ simple interest $\quad R =$ interest rate as a decimal (or fraction)

$P =$ principal $\quad T =$ time periods

e.g. If $3000 is invested for seven years at 6% p.a. simple interest, how much will it be worth after seven years?

$$I = PRT$$

$$I = (3000)(0.06)(7)$$

$$= 1260$$

$\therefore$ Investment is worth $4260 after 7 years

Compound Interest

$$\boxed{A_n = PR^n}$$

Note: *general term of a geometric series*

$A_n =$ amount after n time periods $\qquad P =$ principal

$R = 1 +$ interest rate as a decimal(or fraction)

$n =$ time periods

Note: *interest rate and time periods must match the compounding time*

e.g. If $3000 is invested for seven years at 6% p.a, how much will it be worth after seven years if;

a) compounded annually?

$$A_n = PR^n$$
$$A_7 = 3000(1.06)^7$$
$$A_7 = 4510.89$$

$\therefore$ Investment is worth $4510.89 after 7 years

b) compounded monthly?

$$A_n = PR^n$$
$$A_{84} = 3000(1.005)^{84}$$
$$A_{84} = 4561.11$$

$\therefore$ Investment is worth $4561.11 after 7 years

Depreciation

$$\boxed{A_n = PR^n}$$

$A_n =$ amount after n time periods $\qquad P =$ principal

$R = 1 -$ depreciation rate as a decimal(or fraction)

$n =$ time periods

Note: *depreciation rate and time periods must match the depreciation time*

e.g. An espresso machine bought for \$15000 on 1st January 2001 depreciates at a rate of 12.5%p.a.

a) What will its value be on 1st January 2010?

$$A_n = PR^n$$

$$A_9 = 15000(0.875)^9$$

$$A_9 = 4509.87$$

$\therefore$ Machine is worth \$4509.87 after 9 years

b) During which year will the value drop below 10% of the original cost?

$$A_n = PR^n$$

$$15000(0.875)^n < 1500$$

$$(0.875)^n < 0.1$$

$$\log(0.875)^n < \log 0.1$$

$$n \log 0.875 < \log 0.1$$

$$n > \frac{\log 0.1}{\log 0.875}$$

$$n > 17.24377353$$

$\therefore$ during the 18th year (i.e. 2018) its value will drop to 10% the original cost

Investing Money by Regular Instalments

2002 HSC Question 9b)

A superannuation fund pays an interest rate of 8.75% p.a. which compounds annually. Stephanie decides to invest \$5000 in the fund at the beginning of each year, commencing on 1 January 2003. **(4)**

What will be the value of Stephanie's superannuation when she retires on 31 December 2023?

$A_{21} = 5000(1.0875)^{21}$ amount invested for 21 years

$A_{20} = 5000(1.0875)^{20}$ amount invested for 20 years

$A_{19} = 5000(1.0875)^{19}$ amount invested for 19 years

$\vdots$

$\vdots$

$A_{1} = 5000(1.0875)^{1}$ amount invested for 1 year

$$\text{Amount} = 5000(1.0875)^{21} + 5000(1.0875)^{20} \ldots + 5000(1.0875)$$

$$a = 5000(1.0875), r = 1.0875, n = 21$$

$$= S_{21}$$

$$= \frac{5000(1.0875)\left(1.0875^{21} - 1\right)}{0.0875}$$

$$= \underline{\$299604.86}$$

c*) Find the year when the fund first exceeds $200000.

$$\text{Amount} = 5000(1.0875) + 5000(1.0875)^2 + \ldots + 5000(1.0875)^n$$

$$= S_n$$

$$\text{i.e } S_n > 200000$$

$$\frac{5000(1.0875)\left(1.0875^n - 1\right)}{0.0875} > 200000$$

$$\left(1.0875^n - 1\right) > \frac{280}{87}$$

$$1.0875^n > \frac{367}{87}$$

$$\log\left(1.0875^n\right) > \log\left(\frac{367}{87}\right)$$

$$n\log(1.0875) > \log\left(\frac{367}{87}\right)$$

$$n > \frac{\log\left(\frac{367}{87}\right)}{\log(1.0875)}$$

$$n > 17.16056585$$

$$\therefore n = 18$$

Thus 2021 is the first year when the fund exceeds \$200000 d*) What annual instalment would have produced $1000000 by 31st December 2020?

$$\text{Amount} = P(1.0875)^{18} + P(1.0875)^{17} + \ldots + P(1.0875)$$

$$a = P(1.0875), r = 1.0875, n = 18$$

$$\text{i.e. } S_{18} = 1000000$$

$$\frac{P(1.0875)\left(1.0875^{18} - 1\right)}{0.0875} = 1000000$$

$$P = \frac{(1000000)(0.0875)}{1.0875\left(1.0875^{18} - 1\right)}$$

$$= 22818.16829$$

An annual instalment of $22818.17 will produce $1000000

Loan Repayments

The amount still owing after *n* time periods is;

$$A_n = (\text{principal plus interest}) - (\text{instalments plus interest})$$

e.g. (i) Richard and Kathy borrow \$20000 from the bank to go on an overseas holiday. Interest is charged at 12% p.a., compounded monthly. They start repaying the loan one month after taking it out, and their monthly instalments are \$300.

a) How much will they still owe the bank at the end of six years?

Initial loan is borrowed for 72 months $= 20000(1.01)^{72}$

1^{st} repayment invested for 71 months $= 300(1.01)^{71}$

2^{nd} repayment invested for 70 months $= 300(1.01)^{70}$

$\vdots$

2^{nd} last repayment invested for 1 month $= 300(1.01)^{1}$

last repayment invested for 0 months $= 300$

Repayments are an investment in your loan

$$A_n = (\text{principal plus interest}) - (\text{instalments plus interest})$$

$$A_{72} = 20000(1.01)^{72} - \left\{300 + 300(1.01) \ldots + 300(1.01)^{70} + 300(1.01)^{71}\right\}$$

$$a = 300, r = 1.01, n = 72$$

$$= 20000(1.01)^{72} - \left\{\frac{a(r^n - 1)}{r - 1}\right\}$$

$$= 20000(1.01)^{72} - \left\{\frac{300(1.01^{72} - 1)}{0.01}\right\}$$

$$= \$9529.01$$

b) How much interest will they have paid in six years?

$$\text{Total repayments} = 300 \times 72$$

$$= \$21600$$

$$\text{Loan reduction} = 20000 - 9529.01$$

$$= \$10470.99$$

$$\therefore Interest = 21600 - 10470.99$$

$$= \$11129.01$$

(ii) *Finding the amount of each instalment*

Yog borrows \$30000 to buy a car. He will repay the loan in five years, paying 60 equal monthly instalments, beginning one month after he takes out the loan. Interest is charged at 9% p.a. compounded monthly.

Find how much the monthly instalment shold be.

Let the monthly instalment be \$M

Initial loan is borrowed for 60 months $= 30000(1.0075)^{60}$

1st repayment invested for 59 months $= M(1.0075)^{59}$

2nd repayment invested for 58 months $= M(1.0075)^{58}$

$\vdots$

2nd last repayment invested for 1 month $= M(1.0075)^{1}$

last repayment invested for 0 months $= M$

$$A_n = (\text{principal plus interest}) - (\text{instalments plus interest})$$

$$A_{60} = 30000(1.0075)^{60} - \left\{M + M(1.0075) + \ldots + M(1.0075)^{58} + M(1.0075)^{59}\right\}$$

$$a = M, r = 1.0075, n = 60$$

$$= 30000(1.0075)^{60} - \left\{\frac{a\left(r^n - 1\right)}{r - 1}\right\}$$

$$= 30000(1.0075)^{60} - \left\{\frac{M\left(1.0075^{60} - 1\right)}{0.0075}\right\}$$

But $A_{60} = 0$

$$\therefore 30000(1.0075)^{60} - \left\{\frac{M\left(1.0075^{60} - 1\right)}{0.0075}\right\} = 0$$

$$M\left\{\frac{\left(1.0075^{60} - 1\right)}{0.0075}\right\} = 30000(1.0075)^{60}$$

$$M = \frac{30000(1.0075)^{60}(0.0075)}{\left(1.0075^{60} - 1\right)}$$

$$\therefore \underline{M = \$622.75}$$

(iii) *Finding the length of the loan*

2005 HSC Question 8c)

Weelabarrabak Shire Council borrowed \$3000000 at the beginning of 2005. The annual interest rate is 12%. Each year, interest is calculated on the balance at the beginning of the year and added to the balance owing. The debt is to be repaid by equal annual repayments of \$480000, with the first repayment being made at the end of 2005.

Let A_n be the balance owing after the nth repayment.

(i) Show that $A_2 = (3\times 10^6)(1.12)^2 - (4.8\times 10^5)(1+1.12)$

Initial loan is borrowed for 2 years $= 3000000(1.12)^2$

1st repayment invested for 1 year $= 480000(1.12)^1$

2nd repayment invested for 0 years $= 480000$

$$A_n = (\text{principal plus interest}) - (\text{instalments plus interest})$$

$$A_2 = (3000000)(1.12)^2 - \{480000 + 480000(1.12)\}$$

$$\underline{A_2 = (3\times 10^6)(1.12)^2 - (4.8\times 10^5)(1+1.12)}$$

(ii) Show that $A_n = 10^6\left\{4-(1.12)^n\right\}$

Initial loan is borrowed for n years	$= 3000000(1.12)^n$
1st repayment invested for n – 1 years	$= 480000(1.12)^{n-1}$
2nd repayment invested for n – 2 years	$= 480000(1.12)^{n-2}$
$\vdots$	
2nd last repayment invested for 1 year	$= 480000(1.12)^1$
last repayment invested for 0 years	$= 480000$

$$A_n = (\text{principal plus interest}) - (\text{instalments plus interest})$$

$$A_n = (3000000)(1.12)^n - (480000)\left\{1 + 1.12 + \ldots + (1.12)^{n-2} + (1.12)^{n-1}\right\}$$

$$a = 480000, r = 1.12, n = n$$

$$= 3000000(1.12)^n - \left\{\frac{a\left(r^n - 1\right)}{r-1}\right\}$$

$$A_n = 3000000(1.12)^n - \left\{ \frac{480000\left(1.12^n - 1\right)}{0.12} \right\}$$

$$= 3000000(1.12)^n - 4000000\left(1.12^n - 1\right)$$

$$= 3000000(1.12)^n - 4000000(1.12)^n + 4000000$$

$$= 4000000 - 1000000(1.12)^n$$

$$= 10^6\left\{4 - (1.12)^n\right\}$$

(iii) In which year will Weelabarrabak Shire Council make the final repayment?

$$A_n = 0$$

$$10^6\left\{4 - (1.12)^n\right\} = 0$$

$$4 - (1.12)^n = 0$$

$$(1.12)^n = 4$$

$$\log(1.12)^n = \log 4$$

$$n \log 1.12 = \log 4$$

$$n = \frac{\log 4}{\log 1.12}$$

$$n = 12.2325075$$

The thirteenth repayment is the final repayment which will occur at the end of 2017

Exercise 7B; 4, 6, 8, 10

Exercise 7C; 1, 4, 7, 9

Exercise 7D; 3, 4, 9, 11