Rates of Change

In some cases two, or more, rates must be found to get the equation in terms of the given variable.

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$$

e.g. (*i*) A spherical balloon is being deflated so that the radius decreases at a constant rate of 10 mm/s.
Calculate the rate of change of volume when the radius of the balloon is 100 mm.

$\frac{dV}{dt} = ?$ $\quad V = \frac{4}{3}\pi r^3$ $\quad \frac{dV}{dt} = \frac{dr}{dt} \cdot \frac{dV}{dr}$ $\quad$ when $r = 100$, $\frac{dV}{dt} = -40\pi(100)^2$

$\frac{dr}{dt} = -10$ $\quad \frac{dV}{dr} = 4\pi r^2$

$$= -10\left(4\pi r^2\right)$$
$$= -40\pi r^2$$

$$= -400000\pi$$

$\therefore$ when the radius is 100mm, the volume is decreasing at a rate of $400000\pi \text{mm}^3/\text{s}$

(*ii*) A spherical bubble is expanding so that its volume increases at a constant rate of $70\text{mm}^3/\text{s}$
What is the rate of increase of its surface area when the radius is 10 mm?

$$\frac{dS}{dt} = ? \qquad \frac{dV}{dt} = 70 \qquad V = \frac{4}{3}\pi r^3 \qquad S = 4\pi r^2$$

$$\frac{dV}{dr} = 4\pi r^2 \qquad \frac{dS}{dr} = 8\pi r$$

$$\frac{dS}{dt} = \frac{dV}{dt} \cdot \frac{dS}{dr} \cdot \frac{dr}{dV}$$

$$= (70)(8\pi r)\left(\frac{1}{4\pi r^2}\right)$$

$$= \frac{140}{r}$$

$$\text{when } r = 10, \frac{dV}{dt} = \frac{140}{10}$$

$$= 14$$

$\therefore$ when radius is 10mm the surface area is increasing at a rate of $14\text{mm}^2/\text{s}$

Exercise 7E; 2, 5, 6, 9, 13*

Exercise 7F; 2, 5, 9, 10, 11*