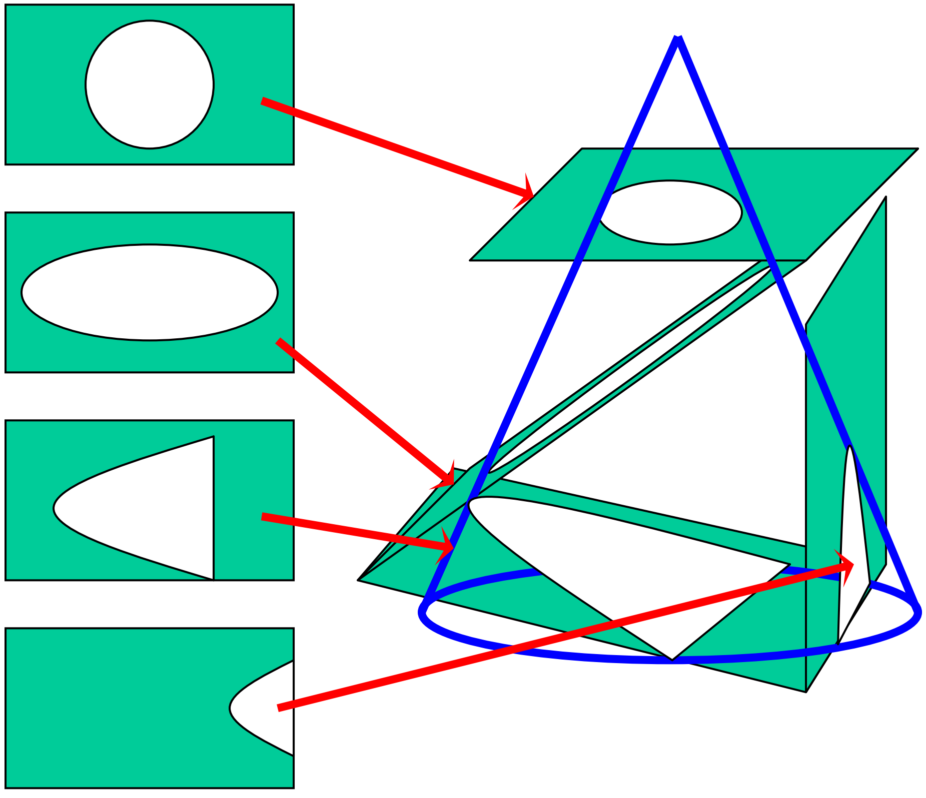
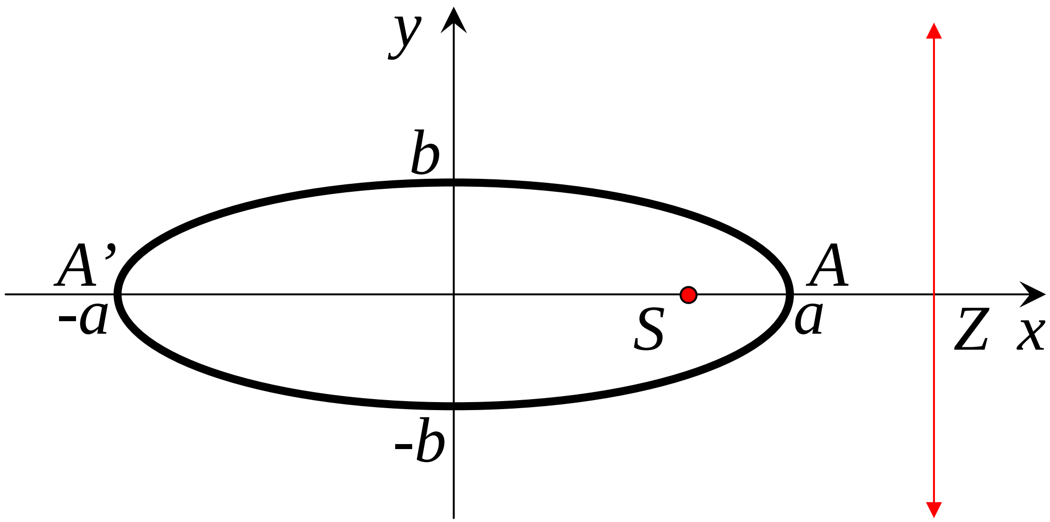
Conics

The locus of points whose distance from a fixed point (**focus**) is a multiple, *e*, (**eccentricity**) of its distance from a fixed line (**directrix**)

$e = 0$	circle
$e < 1$	ellipse
$e = 1$	parabola
$e > 1$	hyperbola



Ellipse ($e < 1$)



$SA = eAZ$ and $SA' = eA'Z$

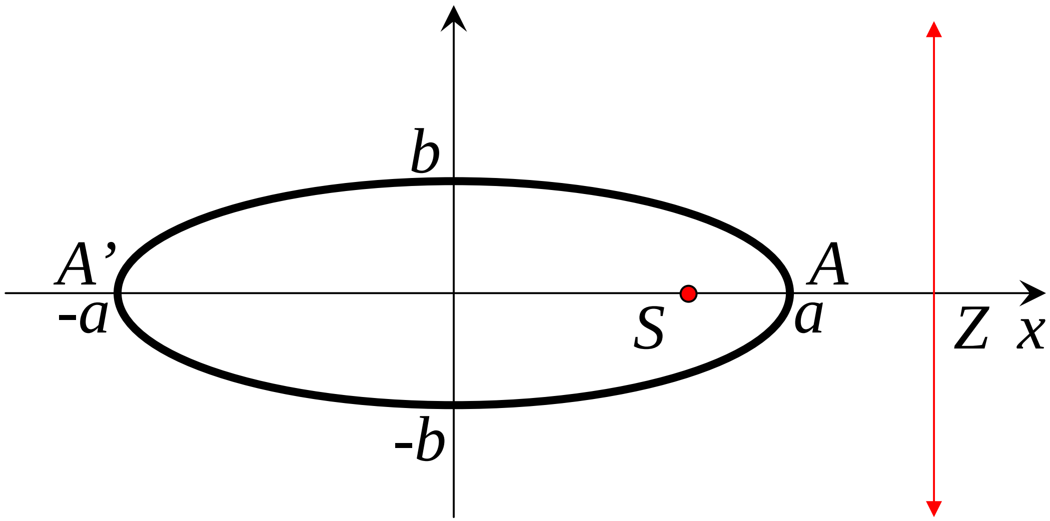
(1) $SA' + SA = 2a$

(2) $SA' - SA = e(A'Z - AZ)$

$= e(AA')$

$= e(2a)$

$= 2ae$



(1) + (2); $2SA' = 2a(1 + e)$
$SA' = a(1 + e)$

(1) - (2); $2SA = 2a(1 - e)$
$SA = a(1 - e)$

Focus

$$OS = OA - SA$$
$$= a - a(1-e)$$
$$= ae$$
$$\therefore S(\pm ae, 0)$$

Directrix

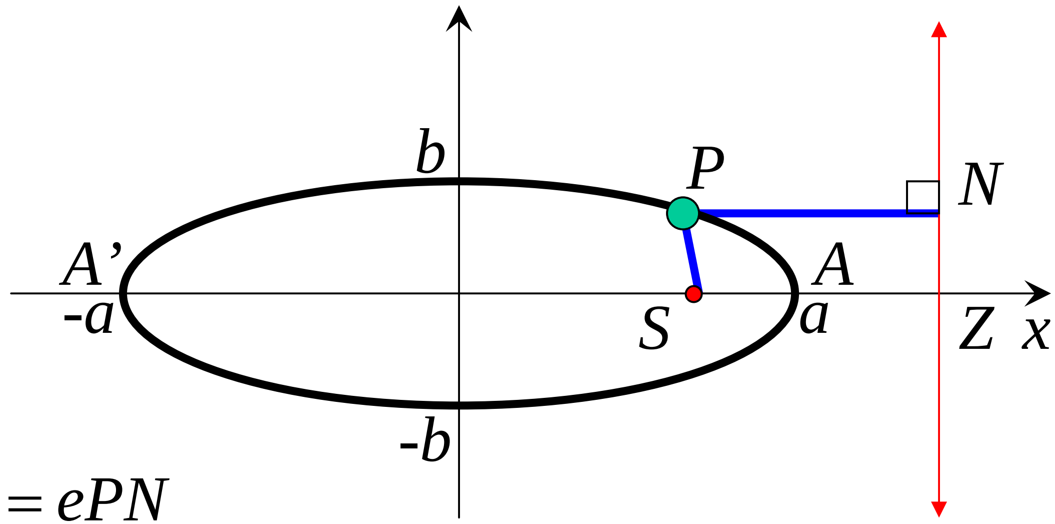
$$OZ = OA + AZ$$
$$= OA + \frac{SA}{e} \qquad (\because SA = eAZ)$$
$$= \frac{ae}{e} + \frac{a(1-e)}{e}$$
$$= \frac{a}{e}$$

$$\therefore \text{directrices } x = \pm \frac{a}{e}$$

$$S(ae,0)$$

$$P(x,y)$$

$$N\left(\frac{a}{e},y\right)$$



$$SP = ePN$$

$$\sqrt{(x-ae)^2+(y-0)^2} = e\sqrt{\left(x-\frac{a}{e}\right)^2+(y-y)^2}$$

$$(x-ae)^2+y^2 = e^2\left(x-\frac{a}{e}\right)^2$$

$$x^2-2aex+a^2e^2+y^2 = e^2x^2-2aex+a^2$$

$$x^2\left(1-e^2\right)+y^2 = a^2\left(1-e^2\right)$$

$$\frac{x^2}{a^2}+\frac{y^2}{a^2\left(1-e^2\right)}=1$$

when $x = 0, y = \pm b$ i.e. $\frac{b^2}{a^2(1-e^2)} = 1$

$$b^2 = a^2(1-e^2)$$

<u>Ellipse</u>: (*a* > *b*)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

where; $b^2 = a^2(1-e^2)$

focus $:(\pm ae, 0)$

directrices $: x = \pm \frac{a}{e}$

e is the eccentricity

<u>major semi-axis</u> = a units

<u>minor semi-axis</u> = b units

<u>Note:</u> If $b > a$

foci on the y axis

$a^2 = b^2(1-e^2)$

focus $:(0, \pm be)$

directrices $: y = \pm \frac{b}{e}$

$$Area = \pi ab$$

e.g. Find the eccentricity, foci and directrices of the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$ and sketch the ellipse showing all of the important features.

$$\frac{x^2}{9}+\frac{y^2}{5}=1$$

$$a^2=9$$

$$a=3$$

$$b^2=5$$

$$a^2(1-e^2)=5$$

$$9(1-e^2)=5$$

$$1-e^2=\frac{5}{9}$$

$$e^2=\frac{4}{9}$$

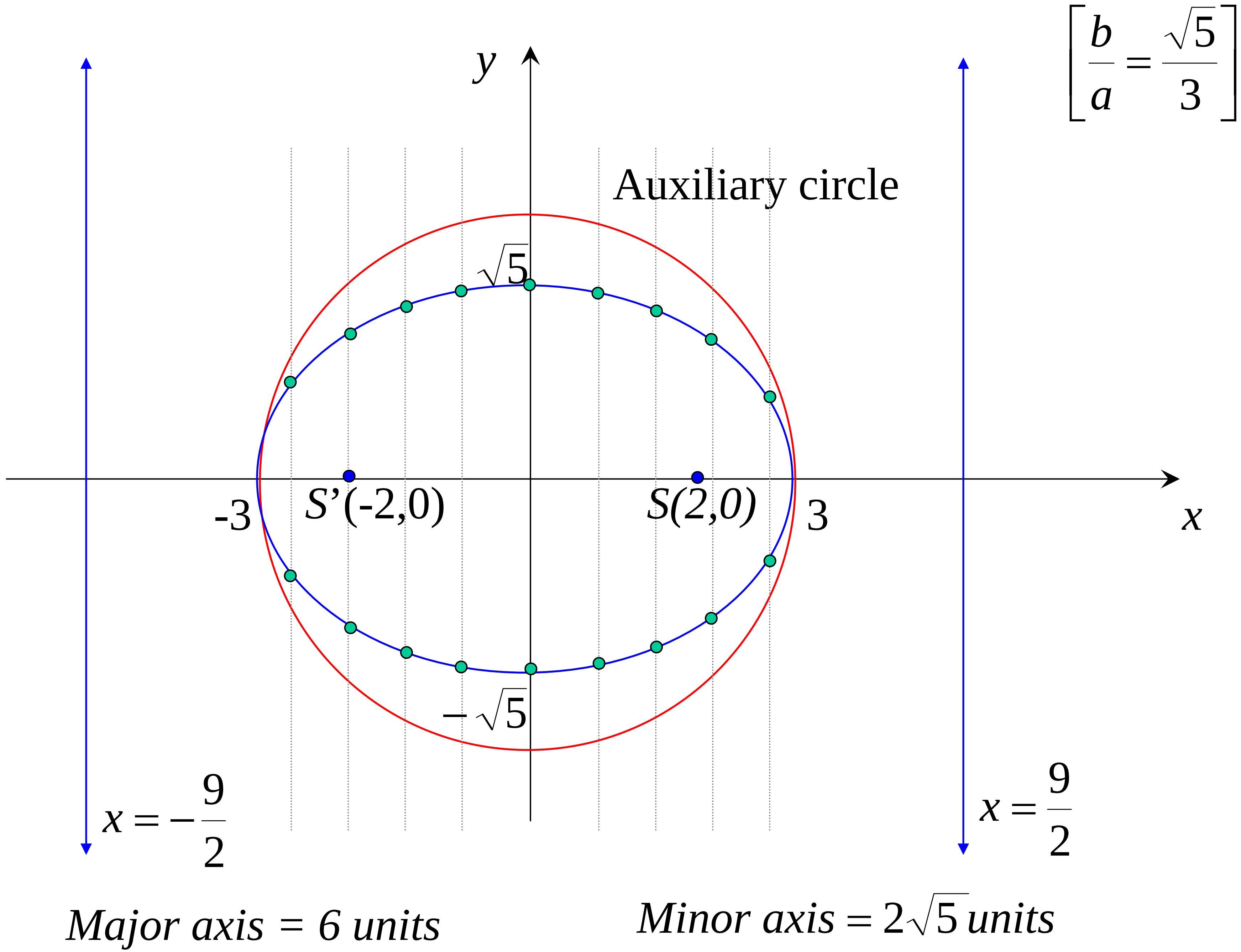
$$e=\frac{2}{3}$$

$\therefore$ eccentricity $=\frac{2}{3}$

foci: $(\pm 2,0)$

directrices: $x=\pm 3\times\frac{3}{2}$

$$x=\pm\frac{9}{2}$$



$\left[\frac{b}{a} = \frac{\sqrt{5}}{3}\right]$
y
Auxiliary circle
$\sqrt{5}$
-3
S'(-2,0)
S(2,0)
3
x
$-\sqrt{5}$
$x = -\frac{9}{2}$
$x = \frac{9}{2}$
Major axis = 6 units
Minor axis = $2\sqrt{5}$ units

$(ii)\quad 9x^2+4y^2+18x-16y-11=0$

$$\frac{x^2+2x}{4}+\frac{y^2-4y}{9}=\frac{11}{36}$$

$$\frac{(x+1)^2}{4}+\frac{(y-2)^2}{9}=\frac{11}{36}+\frac{1}{4}+\frac{4}{9}$$

$$\frac{(x+1)^2}{4}+\frac{(y-2)^2}{9}=1$$

Exercise 6A; 1, 2, 3, 5, 7, 8, 9, 11, 13, 15

$\text{centre}:(-1,2)$

$$b^2=9 \qquad a^2=b^2\left(1-e^2\right)$$

$$b=3 \qquad 4=9\left(1-e^2\right)$$

$$e=\frac{\sqrt{5}}{3}$$

$\text{foci}:\left(-1,2\pm\sqrt{5}\right)$ $\qquad$ $\text{directrices}: y=2\pm\dfrac{9}{\sqrt{5}}$