

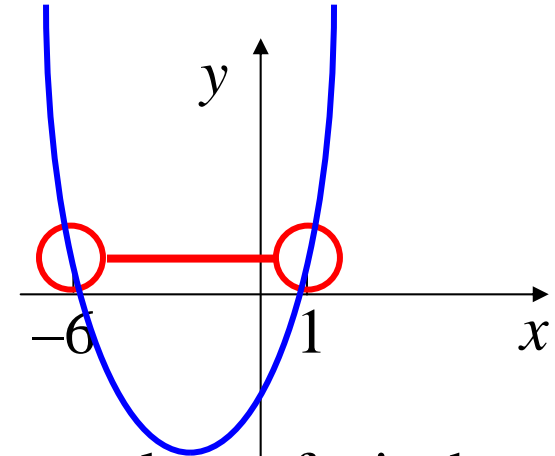
Inequations & Inequalities

1. Quadratic Inequations

e.g. (i) $x^2 + 5x - 6 < 0$

$$(x+6)(x-1) < 0$$

$$\underline{-6 < x < 1}$$



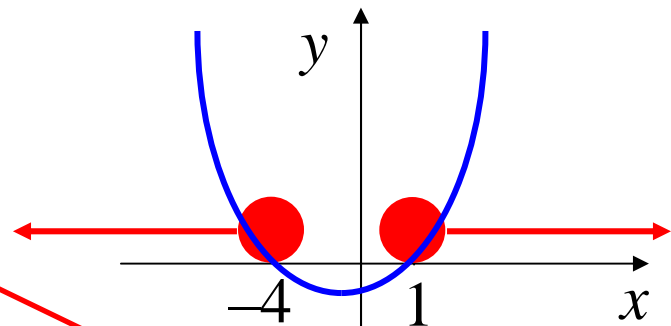
Q: for what values of x is the parabola below the x axis?

(ii) $-x^2 - 3x + 4 \leq 0$

$$x^2 + 3x - 4 \geq 0$$

$$(x+4)(x-1) \geq 0$$

$$\underline{x \leq -4 \text{ or } x \geq 1}$$



Q: for what values of x is the parabola above the x axis?

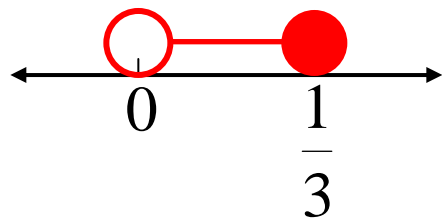
Note: *quadratic inequalities always have solutions in the form ? < x < ? OR x < ? or x > ?*

2. Inequalities with Pronumerals in the Denominator

e.g. (i) $\frac{1}{x} \geq 3$ 1) Find the value where the denominator is zero $x \neq 0$

2) Solve the "equality" $\frac{1}{x} = 3$
 $x = \frac{1}{3}$

3) Plot these values on a number line



4) Test regions

Test $x = -1$ $\frac{1}{-1} \geq 3$ ✗

Test $x = \frac{1}{4}$ $\frac{1}{\frac{1}{4}} \geq 3$ ✓

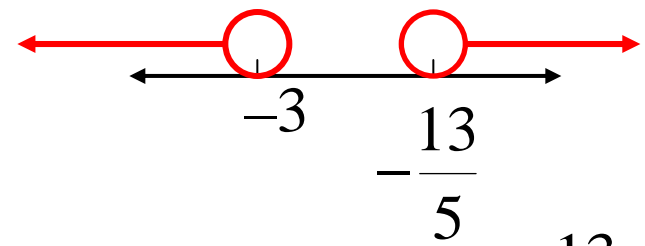
Test $x = 1$ $\frac{1}{1} \geq 3$ ✗

$\therefore 0 < x \leq \frac{1}{3}$

(ii) $\frac{2}{x+3} < 5$

$x+3 \neq 0$
 $x \neq -3$

$\frac{2}{x+3} = 5$
 $2 = 5x + 15$
 $5x = -13$
 $x = -\frac{13}{5}$



$\therefore x < -3$ or $x > -\frac{13}{5}$

3. Proving Inequalities

(I) Start with a known result

If $x - y > y + z$ prove $y < \frac{x - z}{2}$

$$x - y > y + z$$

$$-2y > z - x$$

$$\underline{y < \frac{x - z}{2}}$$

(II) Move everything to the left

Show that if $a \geq 0, b \geq 0$ then $ab(a^2 + b^2) \geq 2a^2b^2$

$$ab(a^2 + b^2) - 2a^2b^2 = ab(a^2 - 2ab + b^2)$$

$$= ab(a - b)^2$$

$$\geq 0$$

$$\underline{\therefore ab(a^2 + b^2) \geq 2a^2b^2}$$

(III) Squares are positive or zero

Show that if a, b and c are positive, then $a^2 + b^2 + c^2 \geq ab + bc + ac$

$$(a - b)^2 \geq 0$$

$$a^2 - 2ab + b^2 \geq 0$$

$$a^2 + b^2 \geq 2ab$$

$$a^2 + c^2 \geq 2ac$$

$$b^2 + c^2 \geq 2bc$$

$$2a^2 + 2b^2 + 2c^2 \geq 2ab + 2bc + 2ac$$

$$a^2 + b^2 + c^2 \geq ab + bc + ac$$

Exercise 3A; 4, 6ace, 7bdf, 8bdf, 9, 11, 12, 13ac, 15, 18bcd, 22, 24