

# *Rational Numbers*

Rational numbers can be expressed in the form  $\frac{a}{b}$  where  $a$  and  $b$  are integers.

# *Irrational Numbers*

Irrational numbers are numbers which are not rational.

All irrational numbers can be expressed as a unique infinite decimal.

e.g. Prove  $\sqrt{2}$  is irrational

**“Proof by contradiction”**

Assume  $\sqrt{2}$  is rational

$\therefore \sqrt{2} = \frac{a}{b}$  where  $a$  and  $b$  are integers with no common factors

$$b\sqrt{2} = a$$

$$2b^2 = a^2$$

Thus  $a^2$  must be divisible by 2

Any square that is divisible by 2 is divisible by 4

Thus  $a^2$  must be divisible by 4

$\therefore 2b^2 = 4k$  where  $k$  is an integer

$$b^2 = 2k$$

So  $a^2$  and  $b^2$  are both divisible by 2 and must have a common factor

However,  $a$  and  $b$  have no common factors

so  $\sqrt{2}$  is not rational

$\therefore \sqrt{2}$  is irrational

**Exercise 2B; 2a, 3, 4 (root 3), 8, 14\***