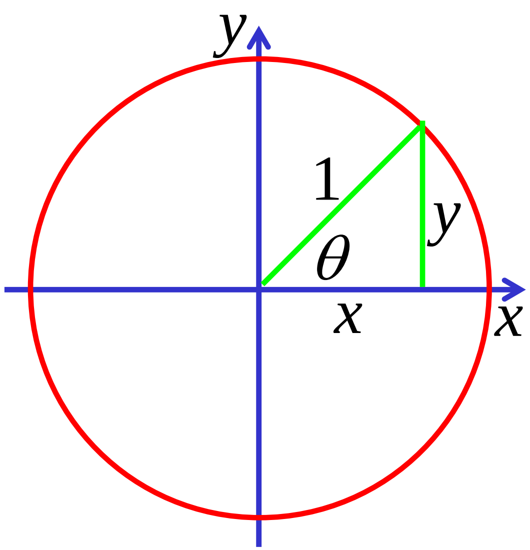
Pythagorean Trig Identities



$$x^2 + y^2 = 1$$

$$\sin\theta = \frac{y}{1} \qquad \cos\theta = \frac{x}{1}$$

$$y = \sin\theta \qquad x = \cos\theta$$

$$\therefore \sin^2\theta + \cos^2\theta = 1$$

Divide by $\sin^2\theta$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$1 + \cot^2\theta = \text{cosec}^2\theta$$

Divide by $\cos^2\theta$

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\sin^2\theta + \cos^2\theta = 1$$

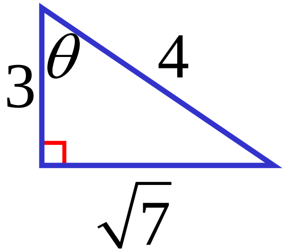
$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \text{cosec}^2\theta$$

In addition:

$$\frac{\sin\theta}{\cos\theta} = \tan\theta$$

e.g. (i) If $\cos\theta = \frac{3}{4}$ and $\tan\theta$ is negative, find $\sin\theta$



4th quadrant

$$\sin\theta = -\frac{\sqrt{7}}{4}$$

(ii) Simplify;

a) $1-\cos^2\theta$

$=1-\left(1-\sin^2\theta\right)$

$=1-1+\sin^2\theta$

$=\sin^2\theta$

b) $\sec^2 A-\tan^2 A$

$=1+\tan^2 A-\tan^2 A$

$=1$

c) $\tan\theta\cos\theta$

$=\dfrac{\sin\theta}{\cos\theta}\times\dfrac{\cos\theta}{1}$

$=\sin\theta$

(iii) Prove $\dfrac{2\sec^2\alpha-2}{4\tan\alpha}=\dfrac{\tan\alpha}{2}$

$$\frac{2\sec^2\alpha-2}{4\tan\alpha}=\frac{2\left(1+\tan^2\alpha\right)-2}{4\tan\alpha}$$

$$=\frac{2+2\tan^2\alpha-2}{4\tan\alpha}$$

$$=\frac{2\tan^2\alpha}{4\tan\alpha}$$

$$=\frac{\tan\alpha}{2}$$

when proving trig identities you can;

1. *start with LHS and prove RHS*
2. *start with RHS and prove LHS*
3. *work on LHS and RHS independently and show they equal the same thing*

NEVER SOLVE LIKE AN EQUATION

Exercise 4E; 1a, 2b, 3ac, 4bd, 5, 7, 9*

Exercise 4F; 3a, 4b, 5c, 6ac, 7bd, 8ac, 10ad, 11acegi, 12bdfhj, 13ac, 14bd, 15ac, 16acegik, 17*a