Symmetry

Odd Functions

$$\boxed{f(-x) = -f(x)}$$

The curve has point symmetry about the origin

$$\left(If \ you \ spin \ it \ 180^\circ \ it \ looks \ the \ same \right)$$

e.g. $y = x^3, \ y = \frac{1}{x}, \ y = x^7 - x^5, \ y = -3x^9 + 2x^7$

Note: "all the powers are odd"

e.g. Prove that $y = x^3 + x^7$ is an odd function

$$f(x) = x^3 + x^7$$

$$\begin{aligned} f(-x) &= (-x)^3 + (-x)^7 \\ &= -x^3 - x^7 \\ &= -\left(x^3 + x^7\right) \\ &= -f(x) \end{aligned}$$

$\therefore$ odd function

Even Functions

$$\boxed{f(-x) = f(x)}$$

The curve has line symmetry about the y axis

$\left(\textit{the y axis is an axis of symmetry}\right)$

e.g. $y = x^2, \quad y = x^2 + 4, \quad y = -3x^6 + 2x^4 - 27x^2$

Note: "all the powers are even"

e.g. Prove that $y = x^2 + 4$ is an even function

$f(x) = x^2 + 4$

$$\begin{aligned} f(-x) &= (-x)^2 + 4 \\ &= x^2 + 4 \\ &= f(x) \end{aligned}$$

$\therefore$ even function

Exercise 3C; 1aceg, 2, 4aceg, 5, 6bdfh, 8adf, 9, 10*