Exponential Growth & Decay

Growth and decay is proportional to population.

$$\frac{dP}{dt} = kP$$

$$\frac{dt}{dP} = \frac{1}{kP}$$

$$t = \frac{1}{k}\int \frac{dP}{P}$$

$$= \frac{1}{k}\log P + c$$

$$kt = \log P + c$$

$$\log P = kt - c$$

$$P = e^{kt-c}$$

$$P = e^{kt} \cdot e^{-c}$$

$$P = Ae^{kt}$$

P = population at time t

A = initial population

k = growth(or decay) constant

t = time

e.g.(i) The growth rate per hour of a population of bacteria is 10% of the population. The initial population is 1000000

a) Show that $P = Ae^{0.1t}$ is a solution to the differential equation.

$$P = Ae^{0.1t}$$

$$\frac{dP}{dt} = 0.1Ae^{0.1t}$$

$$= 0.1P$$

b) Determine the population after $3\frac{1}{2}$ hours correct to 4 significant figures.

when $t = 0, P = 1000000$

$\therefore A = 1000000$

$P = 1000000e^{0.1t}$

when $t = 3.5, P = 1000000e^{0.1(3.5)}$

$= 1419000$

$\therefore$ after $3\frac{1}{2}$ hours there is 1419000 bacteria (*ii*) On an island, the population in 1960 was 1732 and in 1970 it was 1260.

a) Find the annual growth rate to the nearest %, assuming it is proportional to population.

$$\frac{dP}{dt} = kP$$

$$P = Ae^{kt}$$

when $t = 0, P = 1732$

$$\therefore A = 1732$$

$$P = 1732e^{kt}$$

when $t = 10, P = 1260$

$$\text{i.e.} 1260 = 1732e^{10k}$$

$$e^{10k} = \frac{1260}{1732}$$

$$10k = \log\left(\frac{1260}{1732}\right)$$

$$k = \frac{1}{10}\log\left(\frac{1260}{1732}\right)$$

$$k = -0.0318165$$

$\therefore$ growth rate is -3%

b) In how many years will the population be half that in 1960?

when $P = 866$, $866 = 1732e^{kt}$

$$e^{kt} = \frac{1}{2}$$

$$kt = \log\frac{1}{2}$$

$$t = \frac{1}{k}\log\frac{1}{2}$$

$$t = 21.786$$

$\therefore$ In 22 years the population has halved

Exercise 7G; 2, 3, 7, 9, 11, 12