

Asymptotes

Curves always bend towards the asymptotes

Curves never cross a **vertical** asymptote

Curves approach **horizontal** and **oblique** asymptotes as $x \rightarrow \pm\infty$

solve $R(x) = 0$ to find where
(if anywhere) the curve cuts
the horizontal/oblique
asymptote

$$y = \frac{P(x)}{A(x)} = Q(x) + \frac{R(x)}{A(x)}$$

$y = Q(x)$ is the
horizontal/oblique
asymptote

solve $A(x) = 0$ to find
vertical asymptotes

e.g. (i) $y = \frac{(x+3)(x-2)}{(x-1)(x+1)}$

$$x^2 - 1 \overline{) x^2 + x - 6}$$

$$\underline{x^2 \quad -1}$$

$$x - 5$$

$$y = 1 + \frac{x-5}{(x-1)(x+1)}$$

x intercepts: $(-3,0)$, $(2,0)$

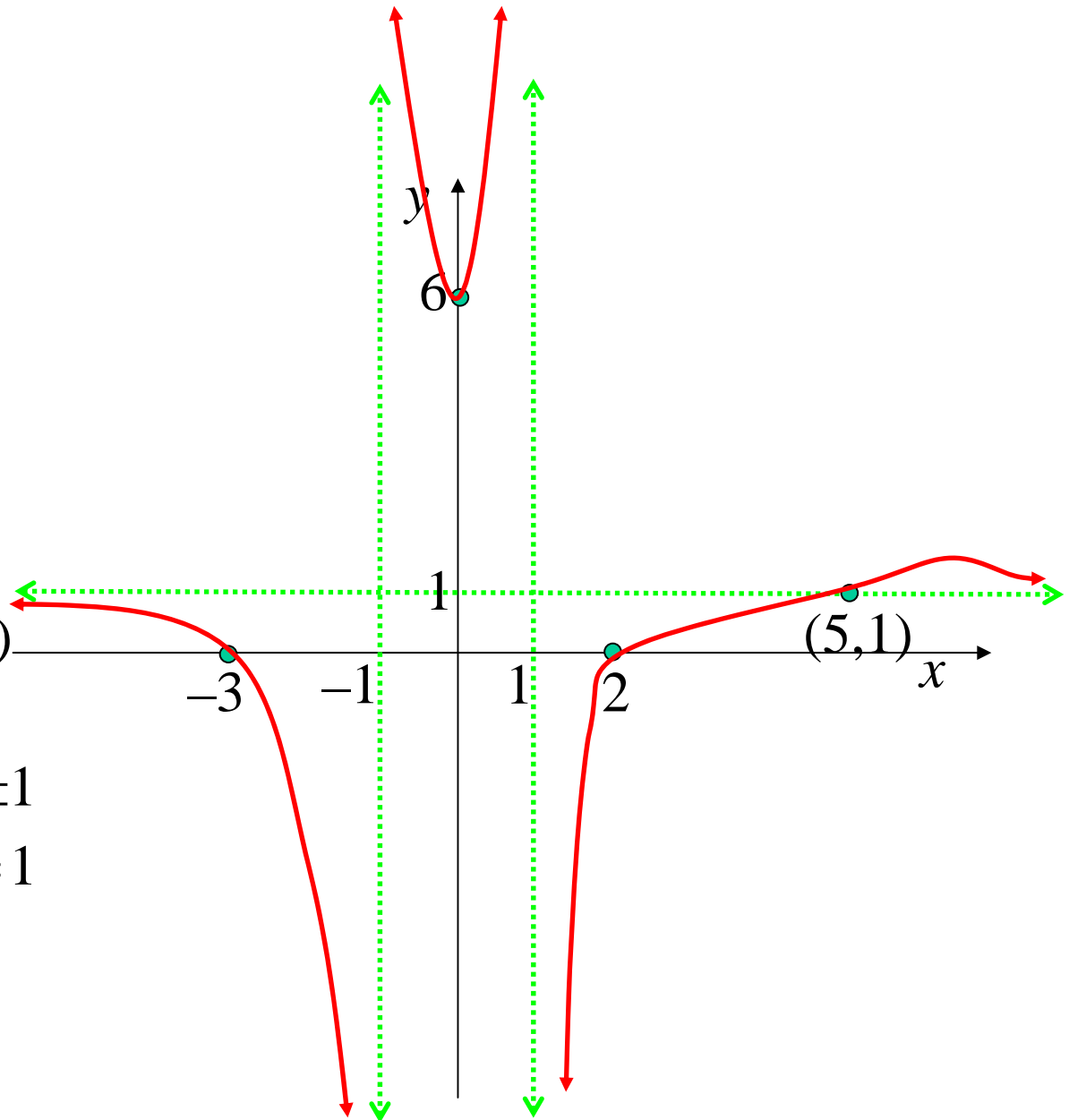
y intercept: $(0,6)$

vertical asymptotes: $x = \pm 1$

horizontal asymptote: $y = 1$

cuts horizontal

asymptote at $x = 5$



e.g. (i) $y = \frac{(x-2)(x-1)(x+1)}{(x+2)(x-3)}$

$$\begin{array}{r}
 x - 1 \\
 \hline
 x^2 - x - 6 \overline{) x^3 - 2x^2 - x + 2} \\
 \underline{x^3 - x^2 - 6x} \\
 -x^2 + 5x + 2 \\
 \underline{-x^2 + x + 6} \\
 4x - 4
 \end{array}$$

$$y = x - 1 + \frac{4x - 4}{(x + 2)(x - 3)}$$

x intercepts: $(-1, 0), (1, 0), (2, 0)$

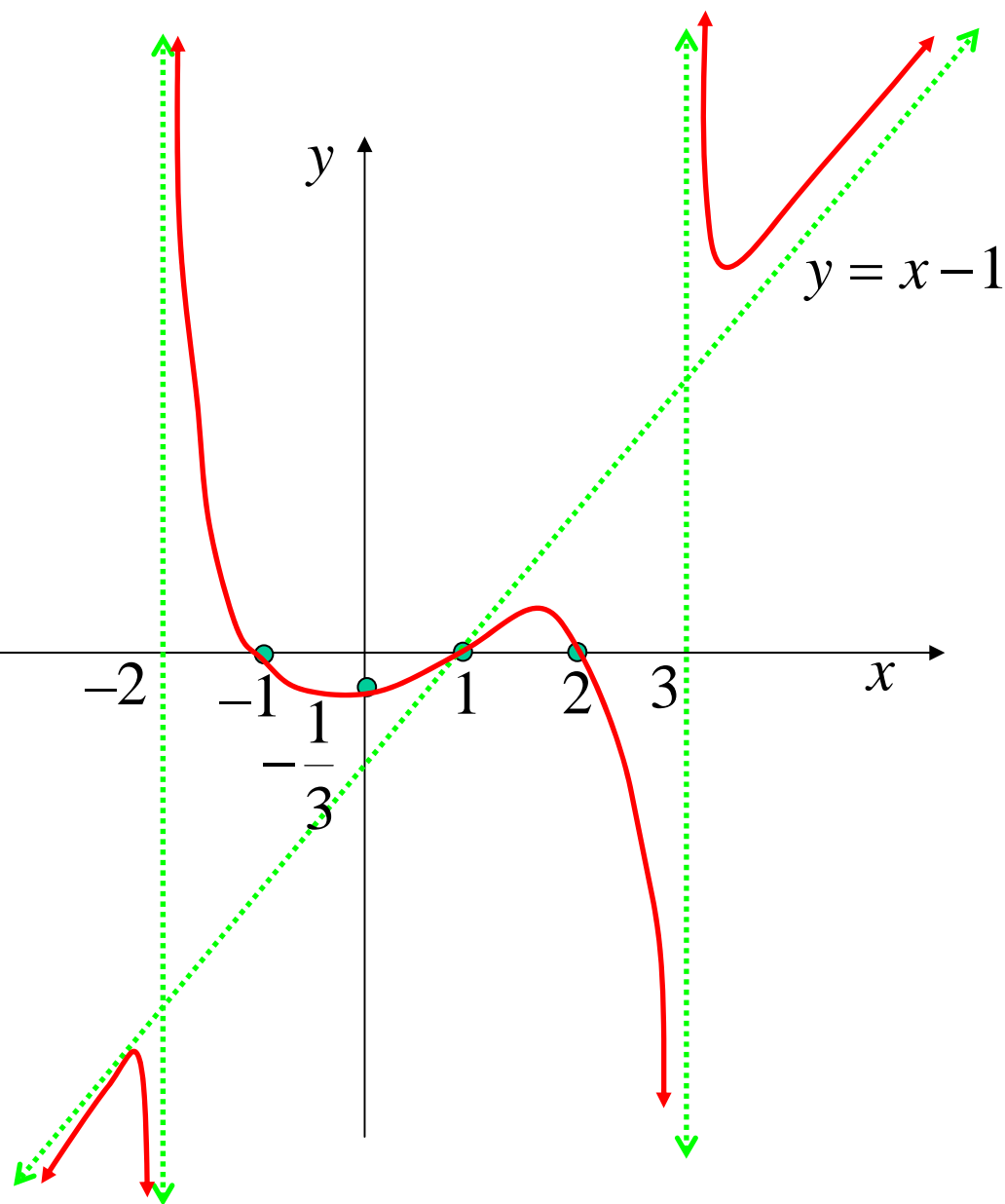
y intercept: $\left(0, -\frac{1}{3}\right)$

vertical asymptotes: $x = -2, 3$

oblique asymptote: $y = x - 1$

cuts horizontal

asymptote at $x = 1$



Exercise 3G; 3, 6, 8ac, 16cf, 17a