Relations & Functions

A **relation** is a set of any ordered pairs that are related in any way.

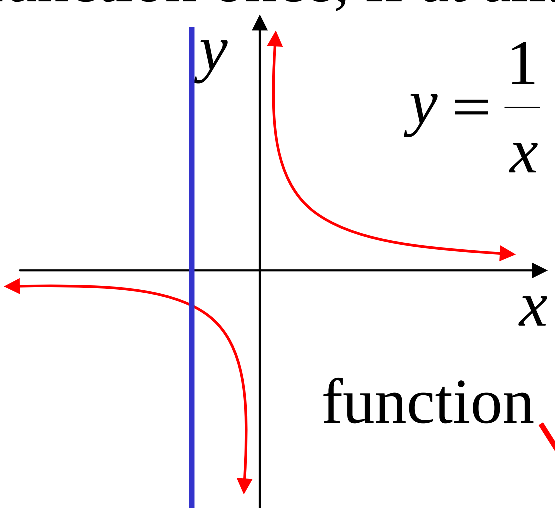
e.g. $x^2 + y^2 = 25$

A **function** is a relation such that for any *x* value, there is a maximum of one *y* value.

e.g. $y = x^2$

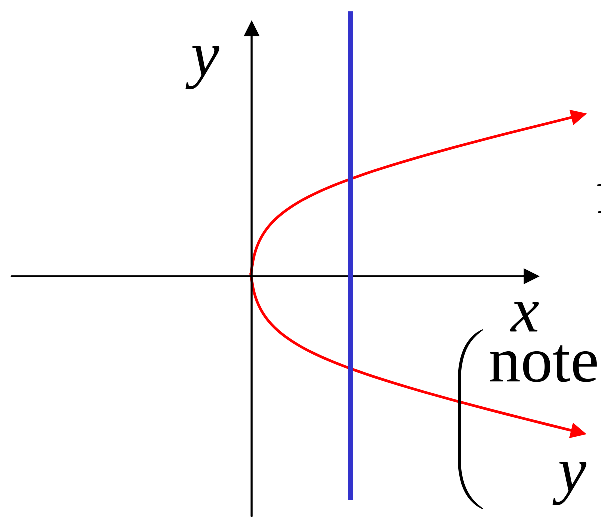
Straight Line Test

If a straight line is drawn parallel to the *y* axis, it will only cross a function once, if at all.



$y = \frac{1}{x}$

function ✓



$x = y^2$

function ✗

(note: actually two functions $y = \sqrt{x}$ and $y = -\sqrt{x}$)

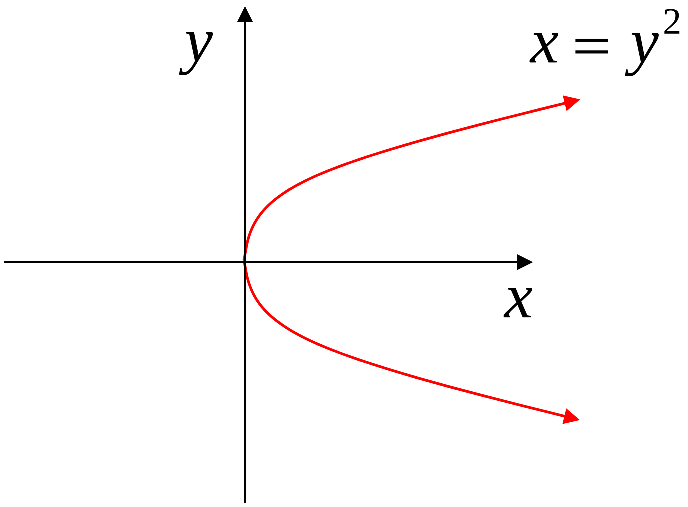
Domain and Range $y = f(x)$

Domain: All possible values of x that can be substituted into the function/relation.

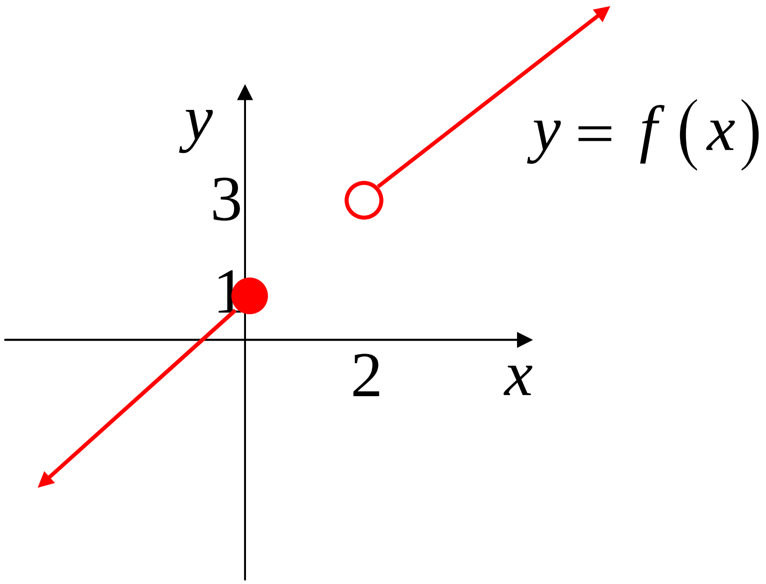
*"Domain is the **INPUT** of the function/relation"*

To find a domain, look for values x could **not** be.

e.g.



domain: $x \geq 0$



domain: $x \leq 0$ and $x > 2$

Things to look for:

1. Fractions: bottom of fraction $\neq$ 0

e.g. $(i)\ y = \dfrac{1}{x}$

$$x \neq 0$$

domain: all real x except $x = 0$

$(ii)\ y = \dfrac{1}{x^2 - 1}$

$$x^2 - 1 \neq 0$$

$$x^2 \neq 1$$

$$x \neq \pm 1$$

domain: all real x except $x = \pm 1$

$(iii)\ y = \dfrac{4x}{x-1} + \dfrac{3}{7-x}$

$$x - 1 \neq 0 \qquad 7 - x \neq 0$$

$$x \neq 1 \qquad x \neq 7$$

domain: all real x except $x = 1$ or 7

2. Root Signs: you can't find the square root of a negative number.

e.g. $(i)\ y = \sqrt{4 - x^2}$

$$4 - x^2 \geq 0$$

$$x^2 \leq 4$$

domain: $-2 \leq x \leq 2$

$(ii)\ y = \sqrt{x+3} - \sqrt{5-x}$

$$x + 3 \geq 0 \qquad 5 - x \geq 0$$

$$x \geq -3 \qquad x \leq 5$$

domain: $-3 \leq x \leq 5$

$(iii)\ y = \dfrac{1}{\sqrt{x+2}}$

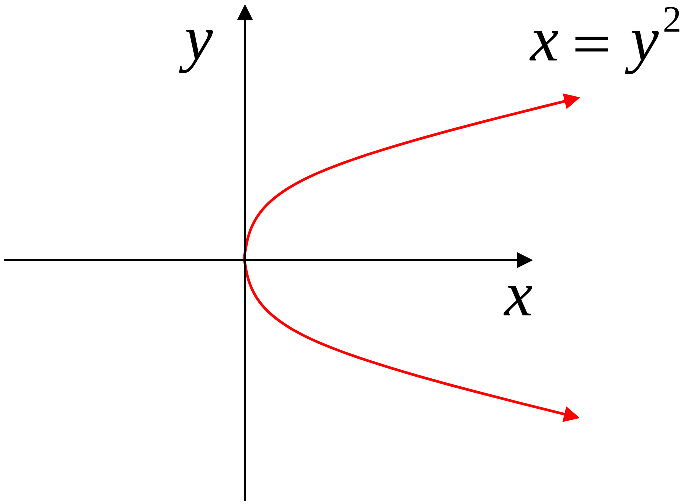
$$x + 2 > 0$$

domain: $x > -2$

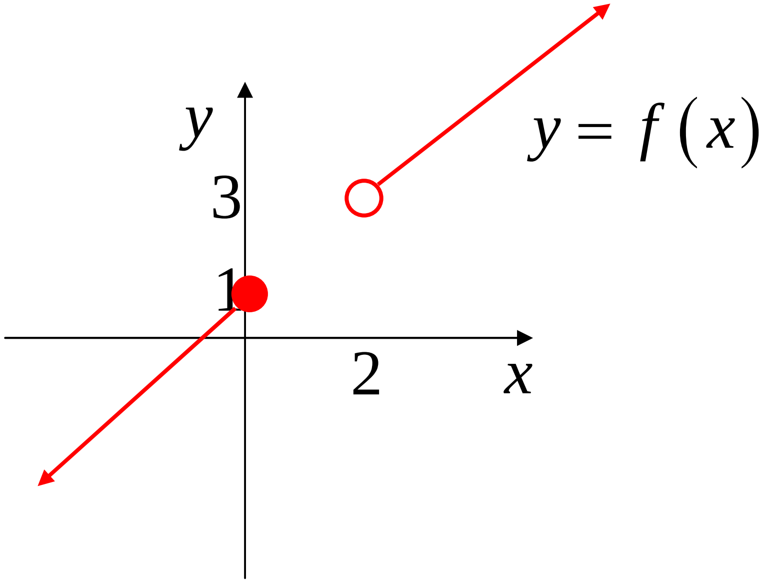
Range: All possible y values obtained by substituting in the domain

*"Range is the **OUTPUT** of the function/relation"*

e.g.



range: all real y



range: $y \leq 1$ and $y > 3$

Things to look for:

1. Maximum/Minimum values:

even powers and absolute values are always $\geq$ 0

e.g. $(i)\ y = x^2$

range: $y \geq 0$

$(ii)\ y = x^2 + 3$

$y \geq 0 + 3$

range: $y \geq 3$

$(iii)\ y = 5 - x^2$

$y \leq 5 - 0$

range: $y \leq 5$

$(iv)\ y = |x + 2|$

range: $y \geq 0$

$(v)\ y = |x + 2| - 5$

$y \geq 0 - 5$

range: $y \geq -5$

2. Restrictions on Domain: sub in endpoints and centre of domain

e.g. $y=\sqrt{4-x^2}$ when $x=2, y=\sqrt{4-2^2}$ when $x=0, y=\sqrt{4-0^2}$

domain: $-2 \le x \le 2$ $=0$ $=2$

range: $0 \le y \le 2$

3. Fractions: If you have a constant on the top of the fraction, fraction $\neq 0$

e.g. $(i)\ y=\frac{1}{x}$

$y \neq 0$

range: all real y except $y=0$

$(ii)\ y=5+\frac{1}{x}$

$y \neq 5+0$

range: all real y except $y=5$

$(iii)\ y=\frac{x+7}{x+4}$

$$
\begin{array}{r}
1 \\
x+4\overline{)\,x+7} \\
\underline{x+4} \\
3
\end{array}
$$

$y=1+\frac{3}{x+4}$

$y \neq 1+0$

range: all real y except $y=1$

Function Notation

e.g. $f(x) = 3x^2 + 4$

$a)\ f(5) = 3(5)^2 + 4$

$= 75 + 4$

$= 79$

$b)\ f(a) = 3a^2 + 4$

$c)\ f(x+h) - f(x) = 3(x+h)^2 + 4 - (3x^2 + 4)$

$= 3x^2 + 6xh + 3h^2 + 4 - 3x^2 - 4$

$= 6xh + 3h^2$

Exercise 2F; 1, 2, 3acdfi, 4begh, 5a, 6, 7a, 8abd, 10abdf, 11aceh, 12bd, 14*