Inverse Relations

If $y = f(x)$ is a relation, then the **inverse relation** obtained by interchanging x and y is $x = f(y)$

e.g. $y = x^3 + x$ inverse relation is $x = y^3 + y$

The domain of the relation is the range of its inverse relation

The range of the relation is the domain of its inverse relation

A relation and its inverse relation are reflections of each other in the line $y = x$.

e.g. $y = x^2$ inverse relation: $x = y^2$

domain: all real x domain: $x \geq 0$

range: $y \geq 0$ range: all real y

Inverse Functions

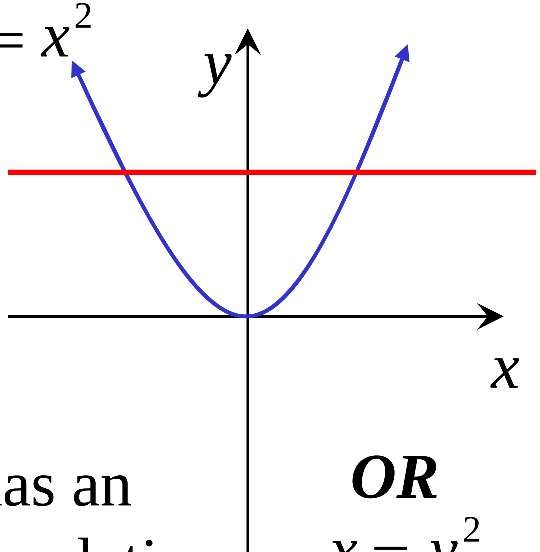
If an inverse relation of a function, is a function, then it is called an **inverse function**.

Testing For Inverse Functions

(1) Use a horizontal line test ***OR***

(2) When $x = f(y)$ is rewritten as $y = g(x)$, $y = g(x)$ is unique.

$(i)\ y = x^2$



Only has an inverse relation

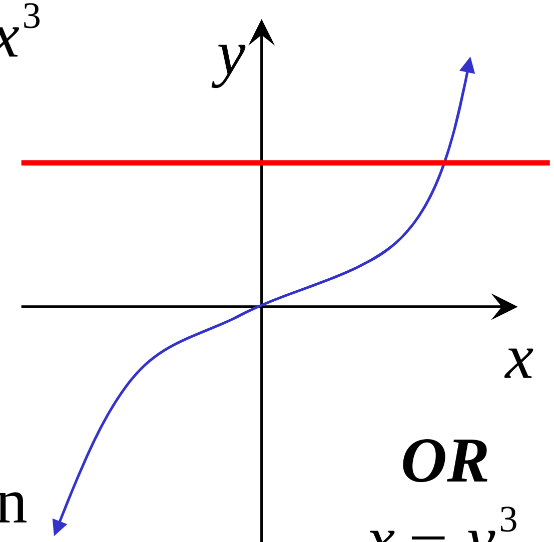
OR

$x = y^2$

$y = \pm\sqrt{x}$

NOT UNIQUE

$(ii)\ y = x^3$



Has an inverse function

OR

$x = y^3$

$y = \sqrt[3]{x}$

UNIQUE

If the inverse relation of $y = f(x)$ is a function, (i.e. $y = f(x)$ has an inverse function), then;

$$f^{-1}(f(x)) = x \qquad \textbf{\textit{AND}} \qquad f\left(f^{-1}(x)\right) = x$$

e.g.

$$f(x) = \frac{x-2}{x+2}$$

$$y = \frac{x-2}{x+2} \Rightarrow x = \frac{y-2}{y+2}$$

$$(y+2)x = y-2$$

$$xy + 2x = y - 2$$

$$(x-1)y = -2x - 2$$

$$y = \frac{2x+2}{1-x}$$

$$f^{-1}(f(x)) = \frac{2\left(\frac{x-2}{x+2}\right)+2}{1-\left(\frac{x-2}{x+2}\right)}$$

$$= \frac{2x-4+2x+4}{x+2-x+2}$$

$$= \frac{4x}{4}$$

$$= x$$ ✓

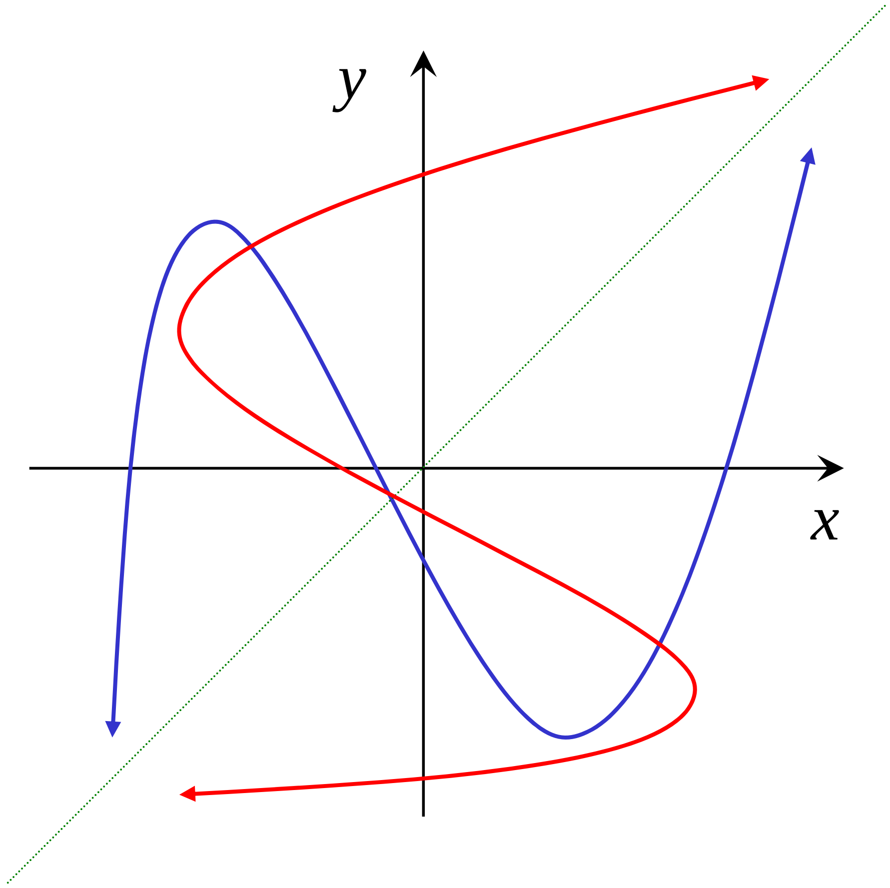
$$f\left(f^{-1}(x)\right) = \frac{\left(\frac{2x+2}{1-x}\right)-2}{\left(\frac{2x+2}{1-x}\right)+2}$$

$$= \frac{2x+2-2+2x}{2x+2+2-2x}$$

$$= \frac{4x}{4}$$

$$= x$$ ✓

(*ii*) Draw the inverse relation



Exercise 2H; 1aceg, 2, 3bdf, 5ac, 6bd, 7ac, 9bde, 10adfhj