

Completing the Square

e.g. (i) $x^2 + 6x - 7 = 0$

$$x^2 + 6x = 7 \quad \longleftarrow \text{move the constant}$$

$$x^2 + 6x + 3^2 = 7 + 3^2 \quad \longleftarrow \text{add half the coefficient of 'x' squared}$$

$$x^2 + 6x + 9 = 16$$

$$(x + 3)^2 = 16 \quad \longleftarrow \text{factorise to a perfect square}$$

$$x + 3 = \pm 4$$

$$x = -3 \pm 4$$

$$\underline{x = -7 \quad \text{or} \quad x = 1}$$

$$(ii) \quad ax^2 + bx + c = 0$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$

$$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2}$$

$$= \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$(iii) x^2 - 6x + 6 = 0$$

$$(x-3)^2 - 3 = 0$$

$$(x-3+\sqrt{3})(x-3-\sqrt{3}) = 0$$

$$x = 3 - \sqrt{3} \quad \text{or} \quad x = 3 + \sqrt{3}$$

Exercise 1I; 1adh, 2ch, 3adg, 4bdfh, 5bdf, 6adg, 7bc, 8*