Inverse Trigonometric Functions

$y = \sin^{-1} x$

$y = \sin x$

Domain: all real x

Range: $-1 \le y \le 1$

NO INVERSE

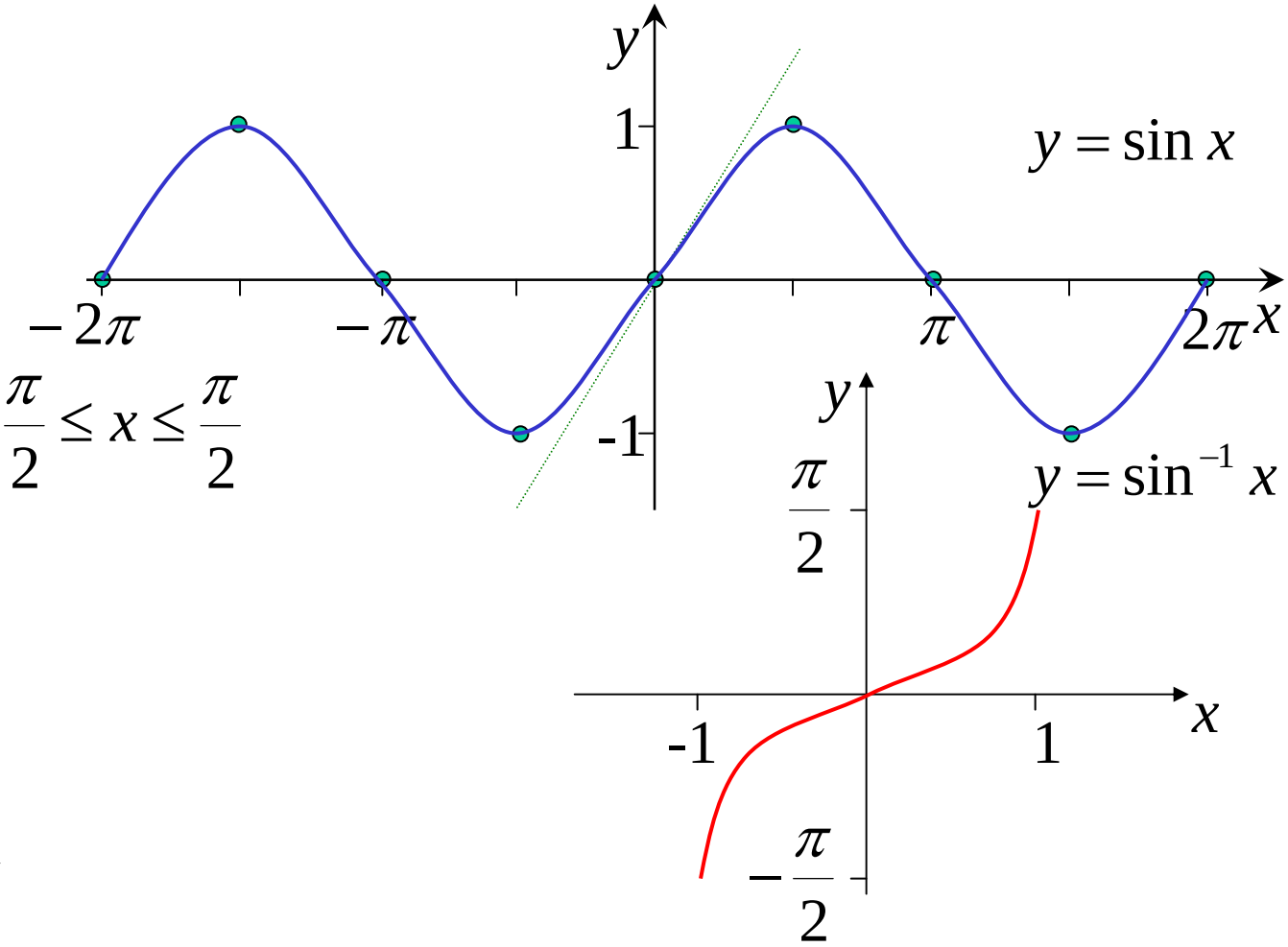
Restricted Domain: $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

Range: $-1 \le y \le 1$

$f^{-1} : y = \sin^{-1} x$

Domain: $-1 \le x \le 1$

Range: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

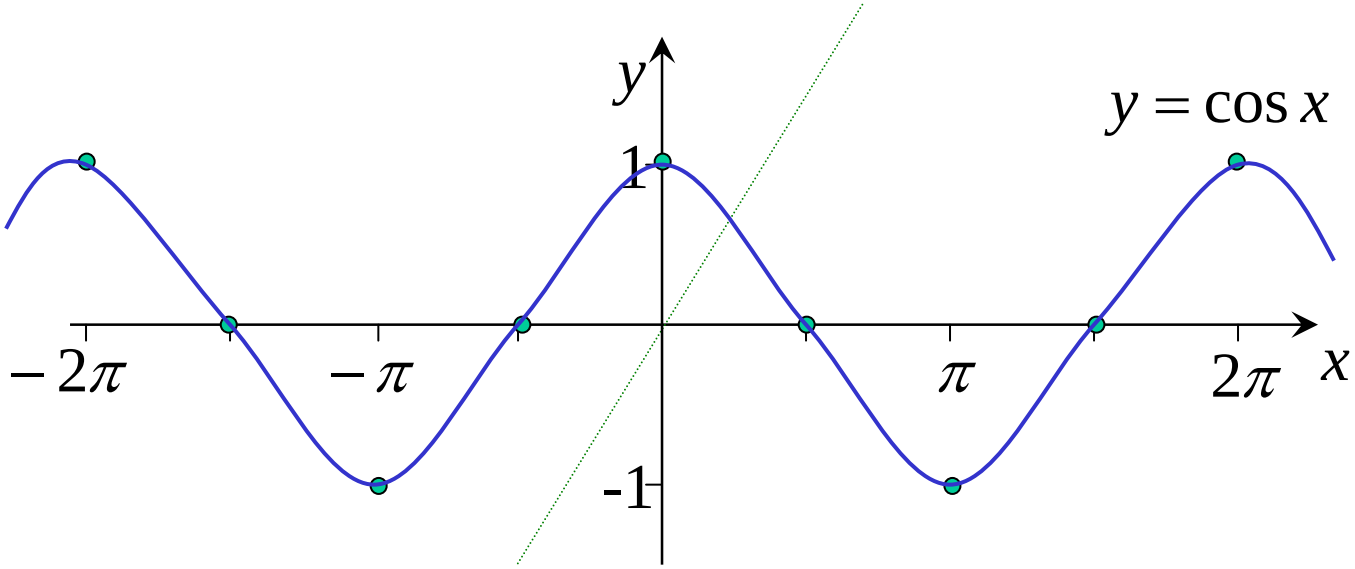


$y = \cos^{-1} x$

$y = \cos x$

Domain: all real x

Range: $-1 \le y \le 1$



NO INVERSE

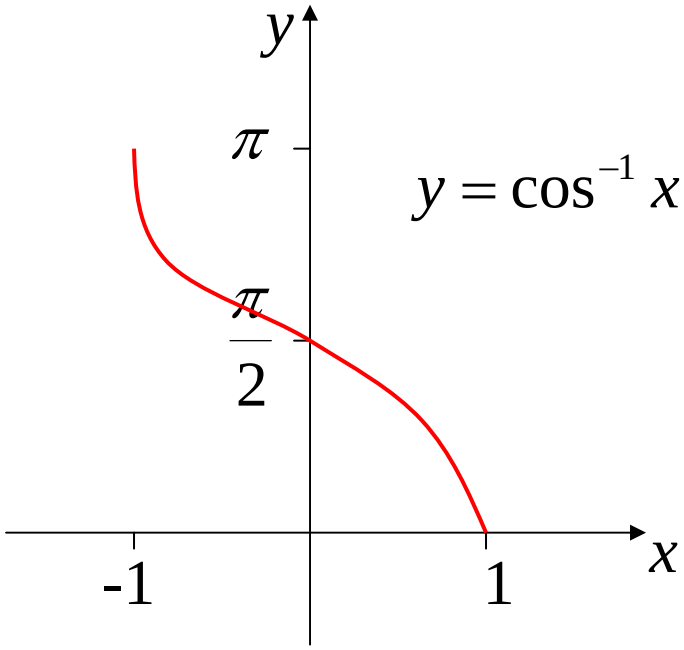
Restricted Domain: $0 \le x \le \pi$

Range: $-1 \le y \le 1$

$f^{-1} : y = \cos^{-1} x$

Domain: $-1 \le x \le 1$

Range: $0 \le y \le \pi$



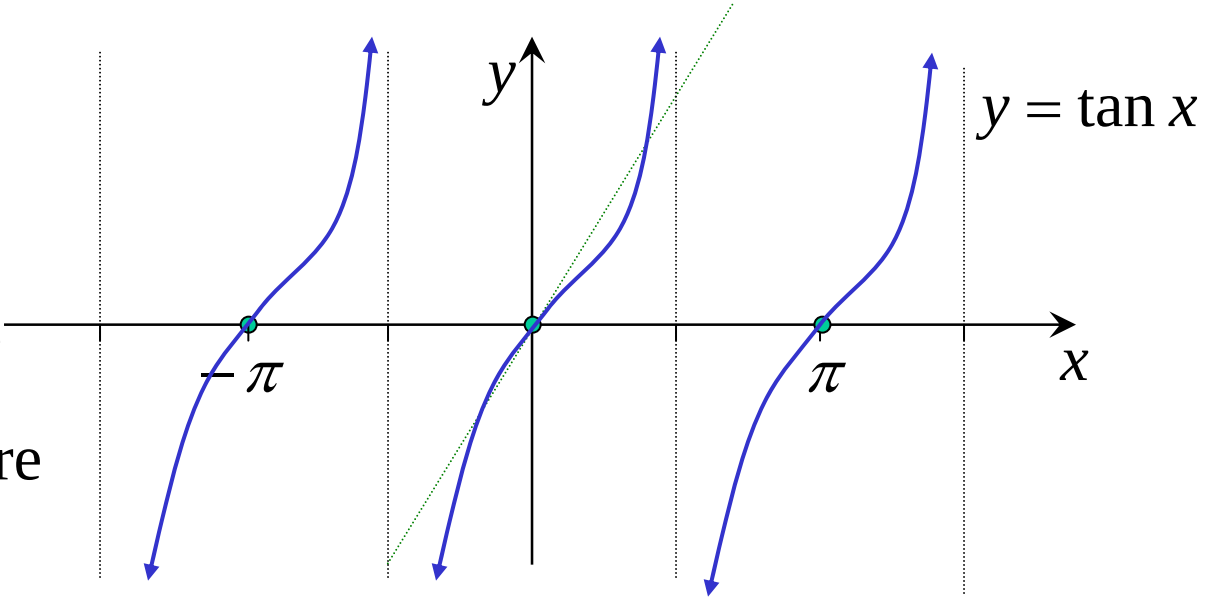
$y = \tan^{-1} x$

$y = \tan x$

Domain: all real x except

$x \neq \frac{\pi}{2} \pm \pi k$ where

k is an integer



Range: all real y

NO INVERSE

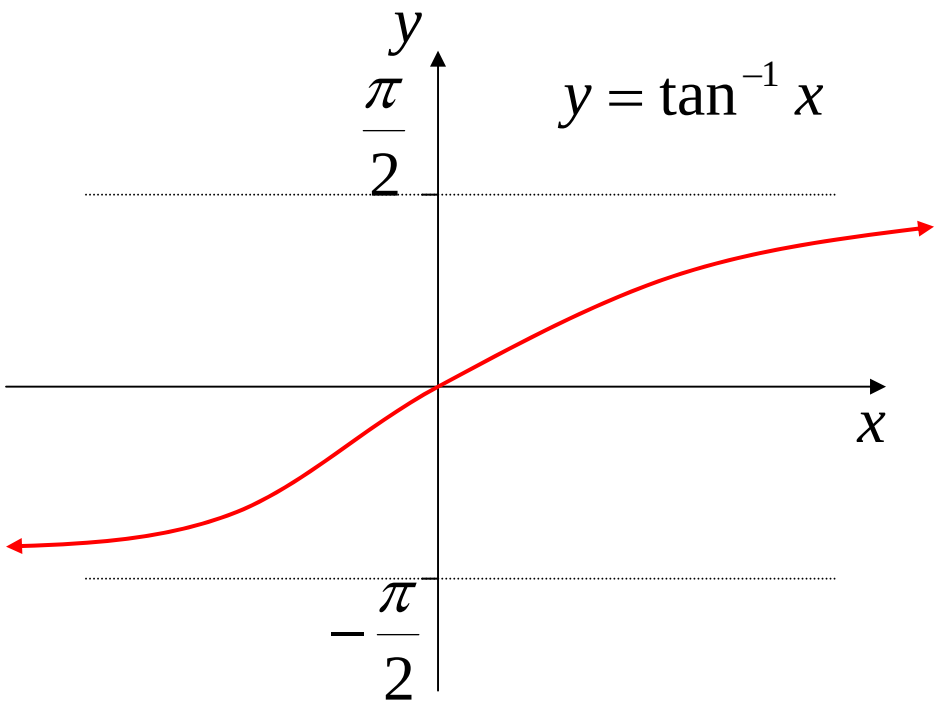
Restricted Domain: $-\frac{\pi}{2} < x < \frac{\pi}{2}$

Range: all real y

$f^{-1} : y = \tan^{-1} x$

Domain: all real x

Range: $-\frac{\pi}{2} < y < \frac{\pi}{2}$



$$\sin^{-1}(-x) = -\sin^{-1} x \qquad \text{(odd function)}$$

$$\cos^{-1}(-x) = \pi - \cos^{-1} x \qquad \left(\text{odd function shifted} \uparrow \frac{\pi}{2}\right)$$

$$\tan^{-1}(-x) = -\tan^{-1} x \qquad \text{(odd function)}$$

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2} \qquad (\angle \text{ sum } \Delta)$$

e.g. Find the exact value of;

$(i)\ \tan^{-1}\sqrt{3} - \tan^{-1} 1$

$$= \frac{\pi}{3} - \frac{\pi}{4}$$

$$= \frac{\pi}{12}$$

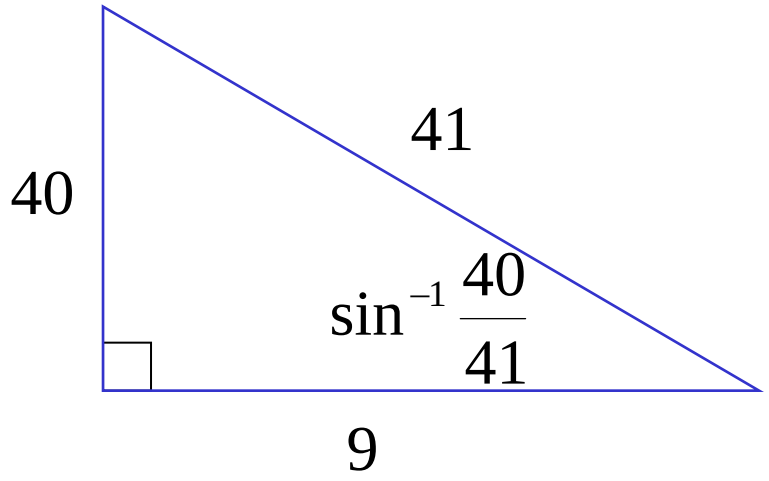
$(ii)\ \sin^{-1}\frac{1}{\sqrt{2}} - \sin^{-1}\left(-\frac{1}{2}\right)$

$$= \frac{\pi}{4} - \left(-\frac{\pi}{6}\right)$$

$$= \frac{5\pi}{12}$$

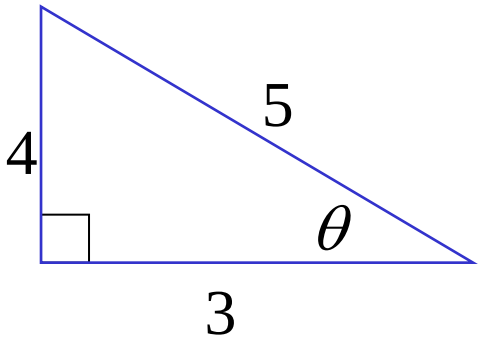
$$(iii)\ \cos\sin^{-1}\frac{40}{41}$$

$$= \frac{9}{41}$$



$$(iv)\ \sin^{-1}\sin\frac{5\pi}{6} = \frac{\pi}{6}$$

$$(v)\ \sin\left(2\cos^{-1}\frac{3}{5}\right) \qquad \text{let } \theta = \cos^{-1}\frac{3}{5}$$



$$= 2\sin\theta\cos\theta$$

$$= 2\left(\frac{4}{5}\right)\left(\frac{3}{5}\right)$$

$$= \frac{24}{25}$$

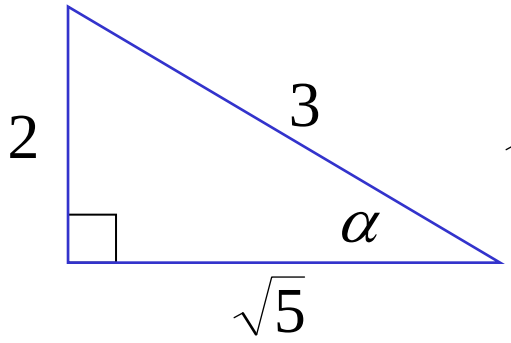
$$(vi)\ \cos^{-1}\left(2\cos\frac{\pi}{3}\right) = \cos^{-1}\left(2\times\frac{1}{2}\right)$$

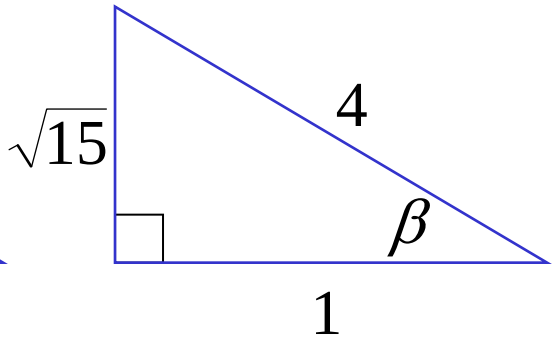
$$= \cos^{-1}1$$

$$= 0$$

$$(vii)\ \tan\left(\sin^{-1}\frac{2}{3}+\cos^{-1}\frac{1}{4}\right)$$

$$\text{let } \alpha=\sin^{-1}\frac{2}{3} \qquad \text{let } \beta=\cos^{-1}\frac{1}{4}$$





$$=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

$$=\frac{\frac{2}{\sqrt{5}}+\sqrt{15}}{1-\left(\frac{2}{\sqrt{5}}\right)\left(\sqrt{15}\right)}$$

$$=\frac{\frac{2+5\sqrt{3}}{\sqrt{5}}}{\frac{\sqrt{5}-2\sqrt{15}}{\sqrt{5}}}$$

$$=\frac{2+5\sqrt{3}}{\sqrt{5}-2\sqrt{15}}$$

Exercise 1B; 1 to 5 ace etc, 6, 7ac, 8ac, 9, 10ac, 11, 12ac, 14 to 20 evens