

Differentiating Inverse Trig

$$y = \sin^{-1} x$$

$$x = \sin y$$

$$\frac{dx}{dy} = \cos y$$

$$\frac{dy}{dx} = \frac{1}{\cos y}$$

$$= \frac{1}{\sqrt{\cos^2 y}}$$

$$= \frac{1}{\sqrt{1 - \sin^2 y}}$$

$$= \frac{1}{\sqrt{1 - x^2}}$$

$$y = \cos^{-1} x$$

$$x = \cos y$$

$$\frac{dx}{dy} = -\sin y$$

$$\frac{dy}{dx} = \frac{-1}{\sin y}$$

$$= \frac{-1}{\sqrt{\sin^2 y}}$$

$$= \frac{-1}{\sqrt{1 - \cos^2 y}}$$

$$= \frac{-1}{\sqrt{1 - x^2}}$$

$$y = \tan^{-1} x$$

$$x = \tan y$$

$$\frac{dx}{dy} = \sec^2 y$$

$$\frac{dy}{dx} = \frac{1}{\sec^2 y}$$

$$= \frac{1}{1 + \tan^2 y}$$

$$= \frac{1}{1 + x^2}$$

In general;

$$y = \sin^{-1} \frac{x}{a}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{a^2 - x^2}}$$

$$y = \cos^{-1} \frac{x}{a}$$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{a^2 - x^2}}$$

$$y = \tan^{-1} \frac{x}{a}$$

$$\frac{dy}{dx} = \frac{a}{a^2 + x^2}$$

$$y = \sin^{-1} f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1 - [f(x)]^2}}$$

$$y = \cos^{-1} f(x)$$

$$\frac{dy}{dx} = \frac{-f'(x)}{\sqrt{1 - [f(x)]^2}}$$

$$y = \tan^{-1} f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{1 + [f(x)]^2}$$

e.g. (i) $y = \sin^{-1} 5x$

$$\frac{dy}{dx} = \frac{5}{\sqrt{1 - 25x^2}}$$

(ii) $y = \cos^{-1} e^x$

$$\frac{dy}{dx} = \frac{-e^x}{\sqrt{1 - e^{2x}}}$$

$$(iii) y = \sin^{-1}\left(\frac{x}{3}\right)$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{9-x^2}}$$

OR

$$y = \sin^{-1}\left(\frac{x}{3}\right)$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{\frac{1}{3}}{\sqrt{1-\frac{x^2}{9}}} \\ &= \frac{\frac{1}{3}}{\frac{1}{3}\sqrt{9-x^2}} \\ &= \frac{1}{\sqrt{9-x^2}} \end{aligned}$$

$$(iv) y = e^{\cos^{-1}x}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{-1}{\sqrt{1-x^2}} e^{\cos^{-1}x} \\ &= \frac{-e^{\cos^{-1}x}}{\sqrt{1-x^2}} \end{aligned}$$

$$(v)y = (\tan^{-1} x)^3$$

$$\frac{dy}{dx} = 3(\tan^{-1} x)^2 \cdot \frac{1}{1+x^2}$$

$$= \frac{3(\tan^{-1} x)^2}{1+x^2}$$

$$(vi)y = x^2 \tan^{-1} x^3$$

$$\frac{dy}{dx} = (x^2) \left(\frac{3x^2}{1+x^6} \right) + (\tan^{-1} x^3)(2x)$$

$$= \frac{3x^4}{1+x^6} + 2x \tan^{-1} x^3$$

**Exercise 1D; 2ace etc, 3bd,
4a, 5, 6a, 9ace etc, 11, 13,
14, 20a***