Integration Involving Inverse Trig

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c \quad \textbf{OR} \quad -\cos^{-1}\left(\frac{x}{a}\right) + c$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$$

e.g.

$$(i)\int \frac{dx}{\sqrt{4 - x^2}}$$

$$= \sin^{-1}\left(\frac{x}{2}\right) + c$$

$$(ii)\int \frac{dx}{9 + x^2}$$

$$= \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + c$$

$$(iii)\int_0^1 \frac{dx}{\sqrt{2 - x^2}} = \left[\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right]_0^1$$

$$= \sin^{-1}\frac{1}{\sqrt{2}} - \sin^{-1} 0$$

$$= \frac{\pi}{4} - 0$$

$$= \frac{\pi}{4}$$

(iv) Find $\frac{d}{dx}\{\sin^{-1} e^x\}$ and hence evaluate $\int_{-\log 2}^{0} \frac{e^x}{\sqrt{1-e^{2x}}}\,dx$

$$\frac{d}{dx}\{\sin^{-1} e^x\} = \frac{e^x}{\sqrt{1-e^{2x}}}$$

$$\therefore \int_{-\log 2}^{0} \frac{e^x}{\sqrt{1-e^{2x}}}\,dx = \left[\sin^{-1} e^x\right]_{-\log 2}^{0}$$

$$= \sin^{-1} e^0 - \sin^{-1} e^{-\log 2}$$

$$= \sin^{-1} 1 - \sin^{-1} \frac{1}{2}$$

$$= \frac{\pi}{2} - \frac{\pi}{6}$$

$$= \frac{\pi}{3}$$

(v) $\int \frac{dx}{\sqrt{4-9x^2}} = \int \frac{dx}{3\sqrt{\frac{4}{9}-x^2}}$

$$= \frac{1}{3}\int \frac{dx}{\sqrt{\frac{4}{9}-x^2}}$$

$$= \frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right) + c$$

Exercise 1E; 2, 3, 4a, 5b, 6 & 7bdf, 9, 12, 13, 16, 17, 20, 22