

Equation of a line through a point and intersection of another two lines

e.g. Find the equation of the line that passes through the intersection of $2x + y + 1 = 0$ and $3x + 5y - 9 = 0$ and the point $(1, 2)$.

$$\begin{array}{rcl} 2x + y + 1 = 0 & \Rightarrow & 10x + 5y = -5 \quad (-) \\ 3x + 5y - 9 = 0 & & \underline{3x + 5y = 9} \\ & & 7x = -14 \\ & & x = -2 \end{array} \quad \begin{array}{l} \therefore 2(-2) + y + 1 = 0 \\ y = 3 \end{array}$$

$$\begin{aligned} m &= \frac{3 - 2}{-2 - 1} \\ &= \frac{1}{-3} \end{aligned}$$

$$\begin{aligned} y - 2 &= -\frac{1}{3}(x - 1) \\ 3y - 6 &= -x + 1 \\ \underline{x + 3y - 7} &= 0 \end{aligned}$$

\therefore the lines intersect at $(-2, 3)$

Alternatively

$$a_1x + b_1y + c_1 + k(a_2x + b_2y + c_2) = 0$$

$$2x + y + 1 + k(3x + 5y - 9) = 0$$

$$(1, 2) : 2(1) + (2) + 1 + k(3(1) + 5(2) - 9) = 0$$

$$5 + 4k = 0$$

$$4k = -5$$

$$k = -\frac{5}{4}$$

$$2x + y + 1 - \frac{5}{4}(3x + 5y - 9) = 0$$

$$8x + 4y + 4 - 15x - 25y + 45 = 0$$

$$7x + 21y - 49 = 0$$

$$\underline{x + 3y - 7 = 0}$$

**Exercise 5F; 2b, 3b, 6b(i),
7ab (i, iii), 9, 10, 13***

Exercise 5G; 2 to 14 evens, 15*