3D Trigonometry

When doing 3D trigonometry it is often useful to redraw all of the faces of the shape in 2D.

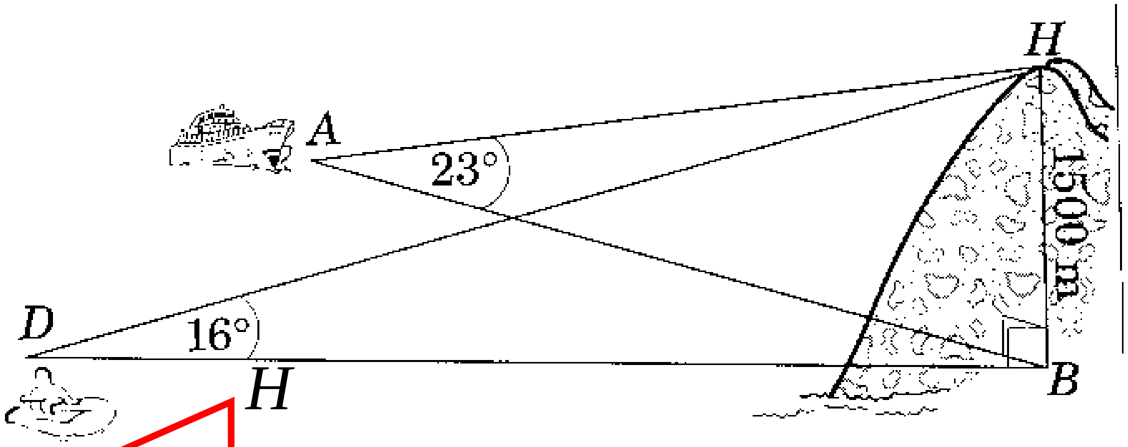
2003 Extension 1 HSC Q7a)

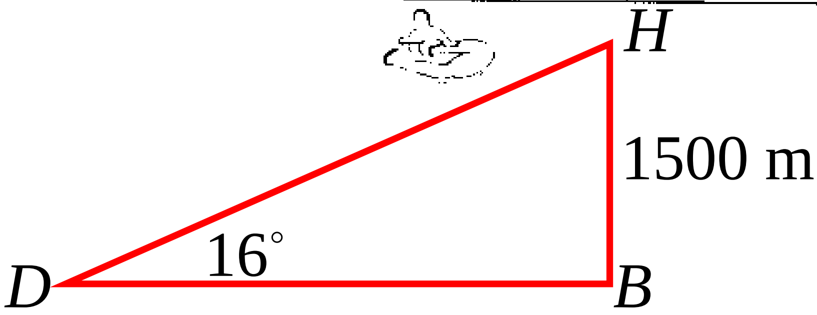
David is in a life raft and Anna is in a cabin cruiser searching for him. They are in contact by mobile phone. David tells Ana that he can see Mt Hope. From David's position the mountain has a bearing of 109°, and the angle of elevation to the top of the mountain is 16°.

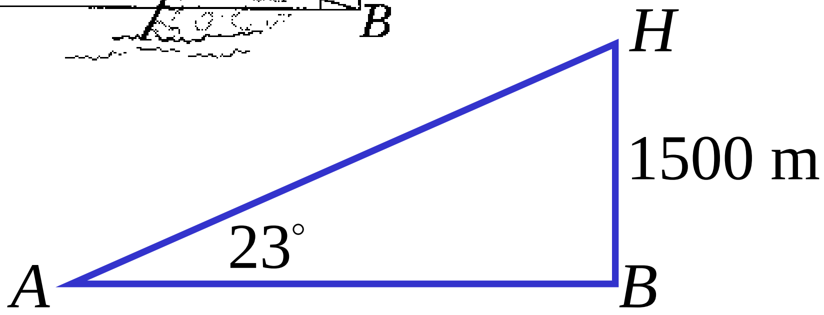
Anna can also see Mt Hope. From her position it has a bearing of 139°, and and the top of the mountain has an angle of elevation of 23°.

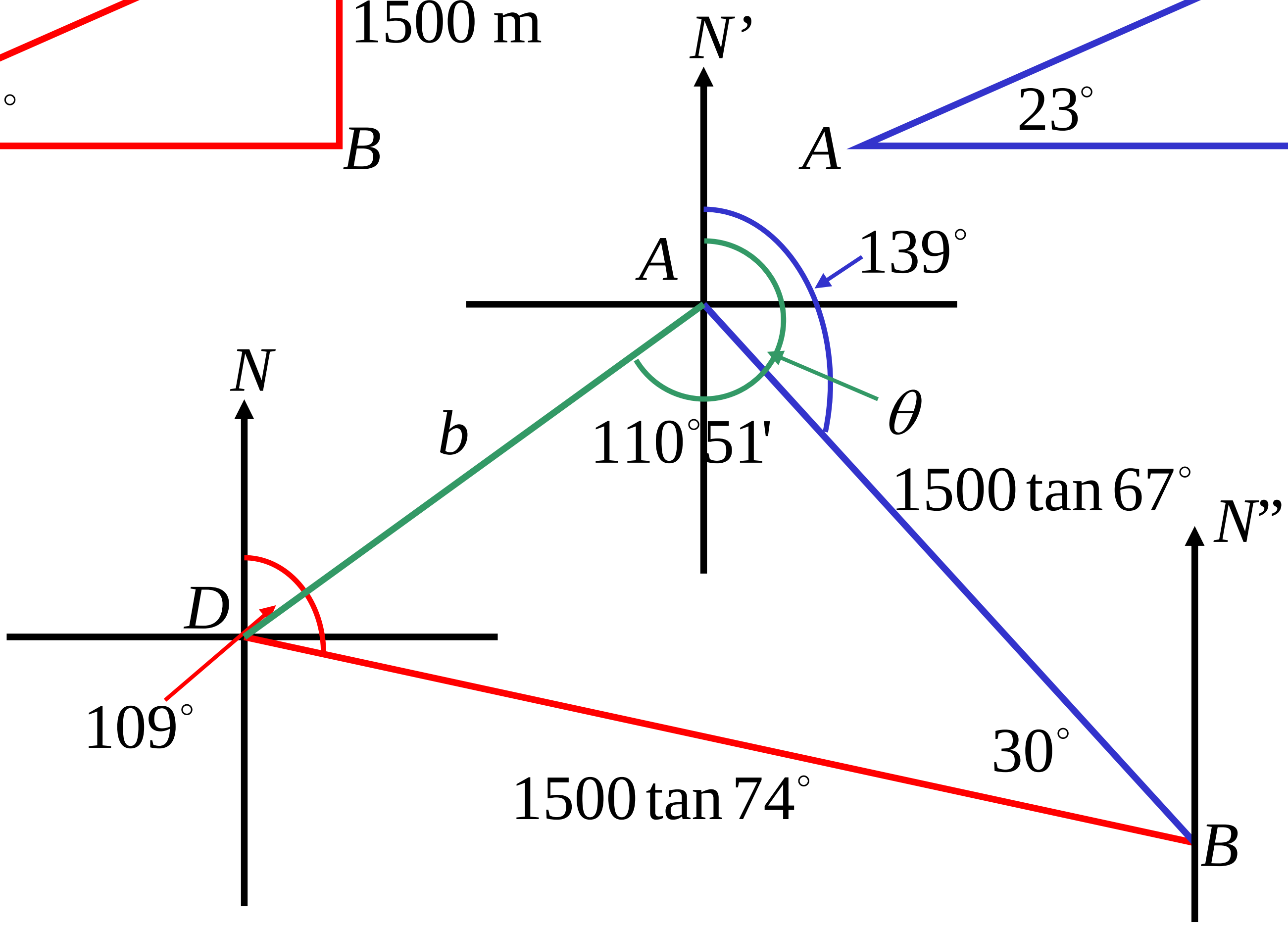
The top of Mt Hope is 1500 m above sea level.

Find the distance and bearing of the life raft from Anna's position.


A
23°
H
1500 m
D
16°
H
B


H
1500 m
D
16°
B


H
1500 m
A
23°
B


N'
A
139°
θ
110°51'
b
N
D
109°
1500 tan 67°
N''
30°
1500 tan 74°
B

$$\frac{BD}{1500} = \tan 74^\circ$$

Similarly;

$$BD = 1500 \tan 74^\circ$$

$$AB = 1500 \tan 67^\circ$$

$$\angle NDB + \angle DBN'' = 180 \qquad \left(\text{cointerior } \angle\text{'s} = 180,\ \text{ND} \parallel \text{N''B}\right)$$

$$109^\circ + \angle DBN'' = 180^\circ$$

$$\angle DBN'' = 71^\circ$$

Similarly;

$$\angle ABN'' = 41^\circ$$

$$\angle ABD = \angle DBN'' - \angle ABN'' \qquad \left(\text{common } \angle\text{'s}\right)$$

$$\therefore \angle ABD = 30^\circ$$

$$b^2 = 1500^2 \tan^2 67^\circ + 1500^2 \tan^2 74^\circ - 2 \times 1500 \tan 67^\circ \times 1500 \tan 74^\circ \cos 30^\circ$$

$$b = 2798.96\ldots$$

$$= 2799 \text{ (to nearest metre)}$$

Anna and David are 2799 m apart.

$$\frac{\sin \angle DAB}{1500 \tan 74^\circ} = \frac{\sin 30^\circ}{b}$$

$$\sin \angle DAB = \frac{1500 \tan 74^\circ \sin 30^\circ}{b}$$

$$\angle DAB = 69^\circ 9' \quad \text{or} \quad 110^\circ 51'$$

If $\angle DAB = 69^\circ 9'$

then $\angle BDA = 80^\circ 51'$

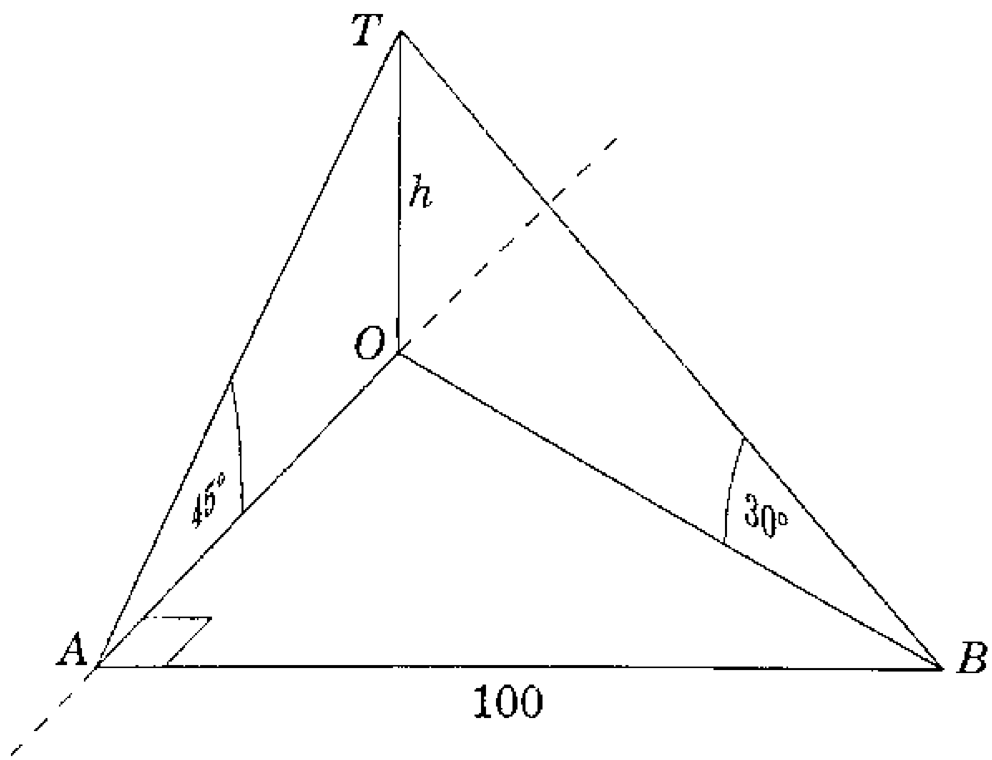
But $\angle DAB > \angle BDA$

$\therefore \angle BDA = 110^\circ 51'$

$\therefore$ The bearing of David from Anna is $249^\circ 51'$

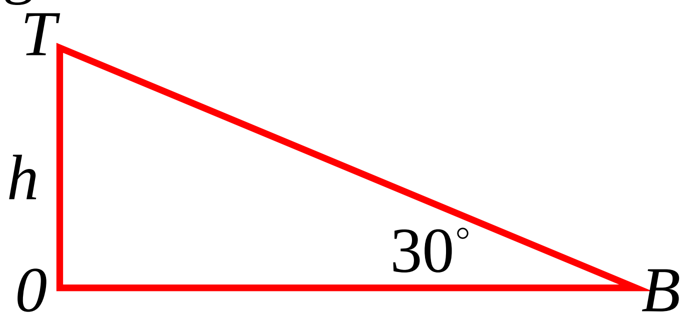
2000 Extension 1 HSC Q3c)

A surveyor stands at point *A*, which is due south of a tower *OT* of height *h* m. The angle of elevation of the top of the tower from *A* is 45°



The surveyor then walks 100 m due east to point *B*, from where she measures the angle of elevation of the top of the tower to be 30°

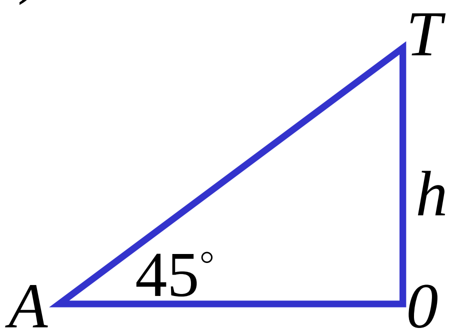
(i) Express the length of OB in terms of h.

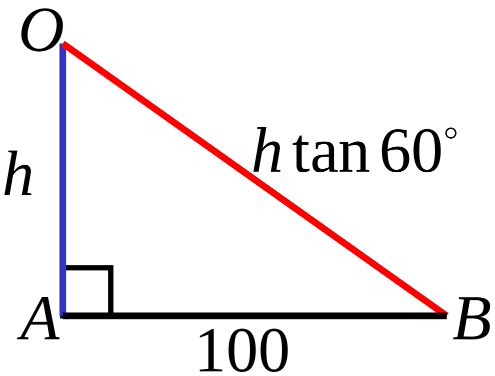


$$\frac{OB}{h} = \tan 60^\circ$$

$$\underline{OB = h\tan 60^\circ}$$

(ii) Show that $h = 50\sqrt{2}$





ΔATO is isosceles

$$\therefore AO = h$$

$$h^2 + 100^2 = h^2 \tan^2 60^\circ$$

$$h^2 + 100^2 = 3h^2$$

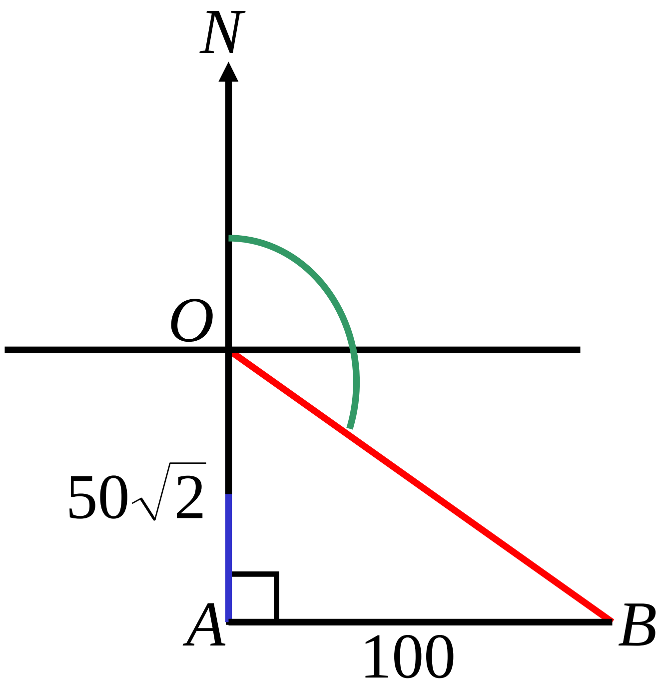
$$2h^2 = 100^2$$

$$h^2 = \frac{100^2}{2}$$

$$h = \frac{100}{\sqrt{2}}$$

$$\underline{h = 50\sqrt{2}}$$

(iii) Calculate the bearing of B from the base of the tower.



$$\tan \angle AOB = \frac{100}{50\sqrt{2}}$$

$$\angle AOB = 54^\circ 44'$$

$$\therefore \text{bearing} = 180^\circ - 54^\circ 44'$$

$$= \underline{125^\circ 16'}$$

Book 2: Exercise 2G odds

Exercise 2H evens