Girraween High School

Year 11 **Mathematics Extension** April, 2005

Task 1 Time: 90 mins

INSTRUCTIONS:
1. Attempt all questions.
2. Write your answers on your own paper.
3. All necessary working must be shown.
4. Marks will be deducted for careless or badly arranged work.

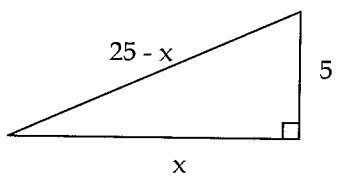
Question 1 (17 marks)

a) (i) $\frac{1}{x-3} > 7$ **3**

(ii) $\frac{5}{2x-3} \leq \frac{3}{4}$ **3**

b) Find k if $(\sqrt{3} - \frac{1}{\sqrt{3}})^2 = k$ **2**

c) Find the value of x. (All lengths in cm) **3**

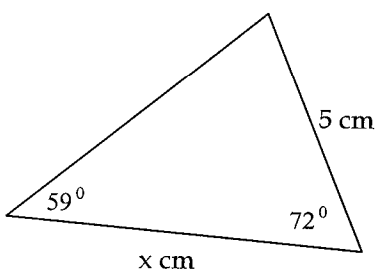


d) Solve for $0^0 \leq \theta \leq 360^0$.

(i) $\sec^2\theta = 2\tan\theta + 4$ (ii) $\operatorname{cosec}\theta = \sec\theta$ **6**

Question 2 (18 marks)

a) Find x, correct to 2 decimal places. **4**



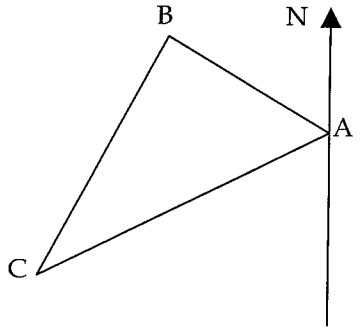
b) $ABCD$ is a parallelogram. $AB = 8cm$, $BC = 5cm$ and $\angle DAB = 120^0$.

(i) Draw a sketch showing this information. **2**

(ii) Calculate the length of the diagonal AC. **3**

(iii) Calculate the area of the parallelogram. **3**

c) Boat B is 30 nautical miles from harbour A
Another boat C is 37 nautical miles from A
and is sailing on a bearing of 245^0. The distance between the boats is 34 nautical miles.



(i) Copy the diagram and indicate the given information. **1**

(ii) Find the size of $\angle BAC$ **3**

(iii) Determine the bearing of boat B from A, correct to the nearest degree. **2**

Question 3 (21 marks)

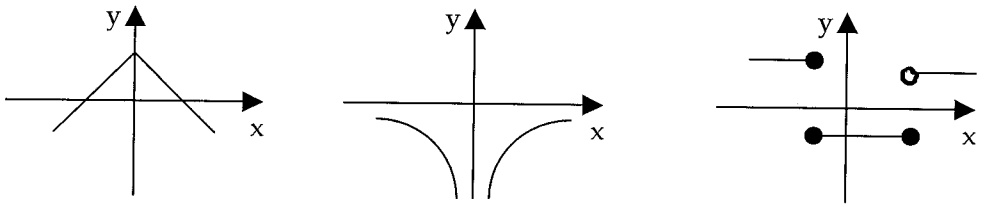
a) If $f(x) = 1 - x^2$, find:

(i) $f(2)$ (ii) $f(-3)$ (iii) $f(2x)$ **4**

b) If $f(x) = 2x - 3$ and $g(x) = 1 + 2x^3$, find $f(3) + g(3)$ **3**

c) State whether these curves represent functions:

(i) (ii) (iii)



3

d) Find the domain of each function:

(i) $y = \dfrac{1}{2-x}$ (ii) $y = \dfrac{1}{x^2 - 1}$ **3**

(iii) $y = \sqrt{x-3}$ (iv) $y = \sqrt{2-x}$ **4**

e) For the function $f(x) = \sqrt{16 - x^2}$.

(i) Explain why the domain is $-4 \le x \le 4$. **2**

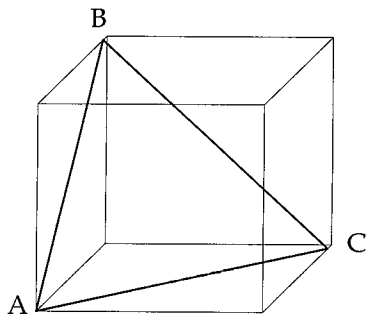
(ii) Find the range of the function. **2**

Question 4 (14 marks)

a) Prove $\dfrac{1 + \cot\theta}{\operatorname{cosec}\theta} - \dfrac{\sec\theta}{\tan\theta + \cot\theta} = \cos\theta$ **3**

b) The shape shown is a cube with sides 6 cm.

Find the size of $\angle ABC$ **3**



c) From a point A, the angle of elevation to the top of a tower due north of it is 20^0. From B, due east of the tower, the angle of elevation is 18^0. A and B are 100 m apart. (Let the top of the tower be point T and the bottom point C)

(i) Draw a sketch to show this information. **2**

(ii) Show the height h of the tower is given by: $h = \dfrac{100}{\sqrt{(\tan^2 72^0 + \tan^2 70^0)}}$ **4**

(iii) Calculate the height h, correct to 1 decimal place. **2**

Question 5 (29 marks)

a) How many 3 digit numbers can be formed from the digits 5, 6, 7, 8 and 9 if no digit is repeated? **2**

b) How many ways can 8 people be arranged in a line if:

(i) there are no restrictions? **1**

(ii) two particular people must be together? **2**

(iii) Graham must be at the beginning and Charlie must be at the end? **2**

c) A committee of 5 must be chosen from 8 boys and 7 girls.
How many different committees can be formed if :

(i) there are no restrictions? **2**

(ii) the committee must have 3 boys and 2 girls? **2**

(iii) there must be more girls than boys on the committee? **3**

(iv) John will not go on the committee if Tracey is on the committee? **3**

d) How many ways can the letters of the word CONNECTION be arranged:

(i) there are no restrictions? **2**

(ii) all the N's must be together? **2**

e) There are 11 different books on a shelf and 3 of them are Maths books. If I select 3 books at random, what is the probability that they will be the 3 Maths books? **2**

f) Ten people are seated around a table.

(i) How many different seating arrangements are possible? **2**

(ii) How many arrangements are possible if Lyall won't sit next to Greg? **2**

(iii) If the 10 people are seated randomly, what is the probability that Lyall will not be sitting next to Greg? **2**

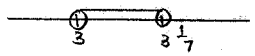
Year 11 Mathematics Extension April 2005 Task 1

Question 1.

a) i) $\frac{1}{x-3} > 7$

- discontinuous $x = 3$
- equal $1 = 7(x-3)$

$x = 3\frac{1}{7}$



$\therefore \quad 3 < x < 3\frac{1}{7}$

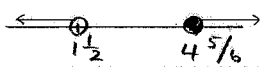
ii) $\frac{5}{2x-3} \leq \frac{3}{4}$

- discontinuous $x = 3/2$
- equal $20 = 3(2x-3)$

$20 = 6x - 9$

$29 = 6x$

$x = 4\,5/6$



$\therefore \quad x < 1\frac{1}{2}, \quad x \geq 4\,5/6$

b) $\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)^2 = 3 - 2 + \frac{1}{3}$

$= 1\frac{1}{3}$

$\therefore \quad k = 1\frac{1}{3}$

c) $(25-x)^2 = 5^2 + x^2$ Pythagoras

$625 - 50x + x^2 = 25 + x^2$

$\therefore \quad 600 = 50x$

$\therefore \quad x = 12$ cm

d) i) $\sec^2\theta = 2\tan\theta + 4$

$1 + \tan^2\theta = 2\tan\theta + 4$

$\tan^2\theta - 2\tan\theta - 3 = 0$

$(\tan\theta - 3)(\tan\theta + 1) = 0$

$\therefore \quad \tan\theta = 3, \quad \tan\theta = -1$

$\therefore \quad \theta = 71^\circ 34', 251^\circ 34', 135^\circ, 315^\circ$

ii) $\operatorname{cosec}\theta = \sec\theta$

$\frac{1}{\sin\theta} = \frac{1}{\cos\theta}$

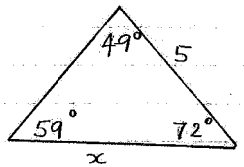
$\therefore \quad 1 = \frac{\sin\theta}{\cos\theta}$

$\therefore \quad 1 = \tan\theta$

$\theta = 45^\circ, 225^\circ$

Question 2

a)

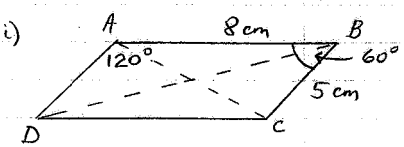


$\frac{x}{\sin 49^\circ} = \frac{5}{\sin 59^\circ}$

$x = \frac{5\sin 49^\circ}{\sin 59^\circ}$

$x = 4.40$ cm

b) i)



ii) $AC^2 = 8^2 + 5^2 - 2\times 8\times 5\times \cos 60^\circ$

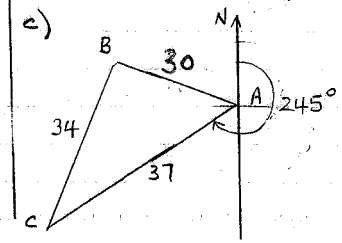
$AC^2 = 49$

$AC = 7$

iii) Area $= \frac{1}{2}\times 8\times 5\times \sin 60^\circ$

$= 34.64\text{ cm}^2$

c)



ii) $\cos\angle BAC = \frac{30^2 + 37^2 - 34^2}{2\times 30\times 37}$

$= 0.50135$

$\therefore \quad \angle BAC = 59.91^\circ$

$\therefore \quad \angle BAC = 60^\circ$

iii) Bearing of B $= 245^\circ + 60^\circ$

$= 305^\circ$

Question 3

a) $f(x) = 1 - x^2$

i) $f(2) = -3$

ii) $f(-3) = -8$

iii) $f(2x) = 1 - (2x)^2$

$= 1 - 4x^2$

b) $f(x) = 2x - 3 \quad g(x) = 1 + 2x^3$

$f(3) + g(3) = (6-3) + (1 + 2\cdot 3^3)$

$= 3 + (1 + 54)$

$= 58$

c) i) function

ii) not a function

iii) function

d) i) all real x, $x \neq 2$

ii) all real x, $x \neq \pm 1$

iii) $x \geq 3$

iv) $2 - x \geq 0$

$\therefore \quad 2 \geq x$

$x \leq 2$

e) $f(x) = \sqrt{16 - x^2}$

i) domain $16 - x^2 \geq 0$

$16 \geq x^2$

$x^2 \leq 16$

so $-4 \leq x \leq 4$

ii) range $0 \leq f(x) \leq 4$

Question 4

a) $\frac{1 + \cot\theta}{\operatorname{cosec}\theta} - \frac{\sec\theta}{\tan\theta + \cot\theta}$

$= \frac{1 + \frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}} - \frac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}}$

$= \frac{\sin\theta + \cos\theta}{1} - \frac{1}{\sin\theta + \frac{\cos^2\theta}{\sin\theta}}$

$= \frac{\sin\theta + \cos\theta}{1} - \frac{\sin\theta}{\sin^2\theta + \cos^2\theta}$

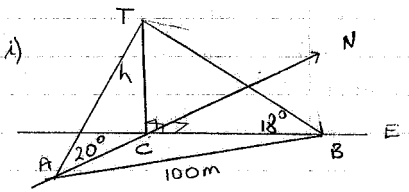
$= \sin\theta + \cos\theta - \sin\theta$

$= \cos\theta$

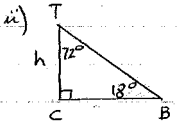
b) Since the solid is a cube, all sides are squares. The diagonals of the squares are equal. ΔABC is equilateral

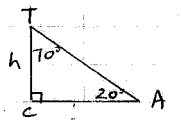
$\therefore \quad \angle ABC = 60^\circ$

c) i)



ii)





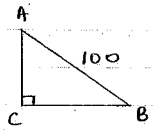
$\tan 72^\circ = \frac{BC}{h}$

$BC = h\tan 72^\circ$

$\tan 70^\circ = \frac{AC}{h}$

$AC = h\tan 70^\circ$

Now in $\triangle ABC$



$$AC^2 + BC^2 = 100^2$$

$$h^2 \tan^2 72^\circ + h^2 \tan^2 70 = 100^2$$

$$h^2 \left[\tan^2 72^\circ + \tan^2 70^\circ \right] = 100^2$$

$$h^2 = \frac{100^2}{\tan^2 72^\circ + \tan^2 70}$$

$$\therefore h = \frac{100}{\sqrt{\tan^2 72^\circ + \tan^2 70^\circ}}$$

iii) $h = \dfrac{100}{4.12562\ldots}$

$= 24.2$ metres.

Question 5

a) $5 \times 4 \times 3 = 60$

b) 8 people

i) $8! = 40\,320$

ii) $7! \times 2 = 10\,080$

iii) G _ _ _ _ _ _ C

$6! = 720$

c) 8B 7G committee 5

i) ${}^{15}C_5 = 3003$

ii) ${}^{8}C_3 \times {}^{7}C_2 = 1176$

iii) committees

5G $= {}^{7}C_5 = 21$

4G + 1B $= {}^{7}C_4 \times {}^{8}C_1 = 280$

3G + 2B $= {}^{7}C_3 \times {}^{8}C_2 = 980$

Total = 1281

iv) ${}^{15}C_5 - {}^{13}C_2$

$= 2717$ different committees

i) $\dfrac{10!}{3! \times 2! \times 2!} = 151\,200$

iii) N's together

$\dfrac{8!}{2!2!} = 10\,080$

e) 11 Books

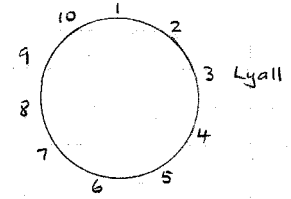
$P(3 \text{ Maths books}) = \dfrac{1}{{}^{11}C_3}$

$= \dfrac{1}{165}$ (0.006)

f)

i) $9! = 362\,880$

ii)



$8 \times 7 \times 7! = 282\,240$

OR $9! - 8! \times 2$

iii) $P(\text{Lyall / Greg not together}) = \dfrac{282\,240}{362\,880}$

$= 0.\dot{7}$