GIRRAWEEN HIGH SCHOOL

HALF YEARLY EXAMINATION

2006

YEAR 11
MATHEMATICS
EXTENSION 1

Time allowed – 90 minutes

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- All necessary working should be shown in every question.

Marks may be deducted for careless or badly arranged work.

- Board-approved calculators may be used.
- Each question attempted is to be returned on a *separate* piece of paper clearly marked Question 1 , Question 2 , etc. Each piece of paper must show your name.
- You may ask for extra pieces of paper if you need them.

QUESTION 1 **(17 Marks)**

(a) Solve for x, $\frac{4}{x-3} < 5$ **3**

(b) Solve for x , $\frac{4x-3}{x+1} \leq 3$ **4**

(c)Solve for x, $|5+2x| = 3x-1$ **3**

(d) Factorise as fully as possible

$a^3 + b^3 + a + b$ **2**

(e) If $f(x) = 2x-5$ and $g(x) = x^2 + 2x + 3$. Find

(i) $f(3) - g(2)$ **3**

(ii) $g(a-1)$ **2**

QUESTION 2 (31 marks)

(a) The number plate on a car has 2 letters followed by 4 numbers. How many different number plates of this type are possible if repetitions are allowed? **2**

(b) (i) Find the number of 4 digit numbers that can be made using the numbers 0 to 9 if each digit can only be used once? **2**

(ii) How many numbers are greater than 6000? **2**

(iii) How many numbers are odd? **2**

(c) A team of 6 is to be chosen from 7 boys and 8 girls.
How many ways can the team be selected if there are

(i) All boys? **2**

(ii) 3 boys and 3 girls ? **2**

(iii) If a particular boy is included? **2**

(iv) More girls than boys? **4**

(v) If two particular girls A and B are not together? **3**

(d) 4 women, 2men and a child sit at a round table. In how many ways can these 7 people be arranged if

(i) If there are no restrictions in the selection ? **1**

(ii) If the child is seated between two men? **2**

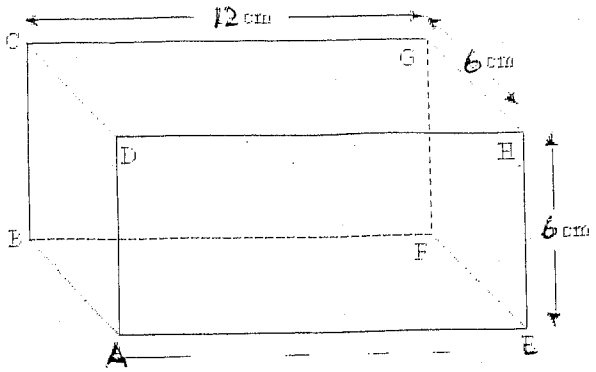
(iii) If the child is seated between two women? **2**

(e) (i) How many distinct 10 letter words can be formed from the letters of the word **BROTHERTON** ? **3**

(ii)Find the probability of the two T's being together. **2**

QUESTION 3 (12 Marks)

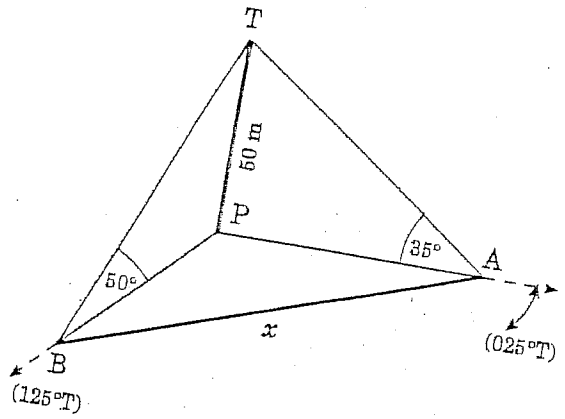
(a)



(i) Find AF **2**

(ii) Find $\angle GAF$ to nearest degree **2**

(b)



PT is an observation tower 50m high. The bearings of two points A and B from P are 025° T and 125° T respectively. The angles of elevation from these points to the top of the tower are 35° and 50° respectively.

(i) Find $\angle APB$ giving a reason **2**

(ii) Show that $AP = 50\cot 35^\circ$ **2**

(iii) Show that $AB^2 = 50^2(\cot^2 35^\circ + \cot^2 50^\circ - 2\cot 35^\circ \cot 50^\circ \cos 100^\circ)$ **3**

(iv) Hence find AB to nearest metre **1**

QUESTION 4 (16 Marks)

(a) Give neat sketches on separate diagrams for

(i) $f(x) = -x^2 + 2$ **2**

(ii) $f(x) = |x - 2|$ **2**

(iii) $f(x) = \frac{1}{x}$ **2**

(b) State the domain and range for

(i) $y = |x - 2|$ (ii) $y = \frac{1}{x + 2}$ (iii) $y = 3^x$ **6**

(c) Is $y = \frac{1}{x}$ a function? Give a reason. **2**

(d) Is $y=x^2+8$ odd, even or neither? Justify your answer. **2**

QUESTION 5 (17 Marks)

(a) Graph $y = \cos x$ $0 \le x \le 360$ **3**

(b) (i) Solve the following equations for $0 \le x \le 360$

(i) $4\cos x + 4\sin^2 x = 5$ **4**

(ii) $\cos 2\theta = \frac{-\sqrt{3}}{2}$ **3**

(c) Prove the identity

$$\frac{\cos A - \tan A \sin A}{\cos A + \tan A \sin A} \equiv 1 - 2\sin^2 A$$ **4**

(d) $\sqrt{5} = 2 + \frac{1}{a}$ show that $a = 4 + \frac{1}{a}$ **3**

Solutions to Year 11 Extension 1 Half Yearly 2006.

Question 1

(a) $\frac{4}{x-3} < 5$

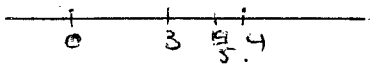
① $x \neq 3$

② $\frac{4}{x-3} = 5$

$4 = 5x - 15$

$5x = 19$

$x = \frac{19}{5}$



Test $x = 0$ $-\frac{4}{3} < 5$ ✓

$x = 3\frac{1}{2}$ $8 < 5$ ×

$x = 5$ $2 < 3$ ✓

$\therefore x > \frac{19}{5}$ or $x < 3$

other methods also valid (3)

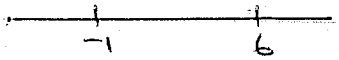
(b) $\frac{4x-3}{x+1} \leq 3$

① $x \neq -1$

② $\frac{4x-3}{x+1} = 3$

$4x - 3 = 3x + 3$

$x = 6$



① $x = -2$ $\frac{-11}{-1} \leq 3$ ×

② $x = 0$ $-3 \leq 3$ ✓

③ $x = 7$ $\frac{25}{8} \leq 3$ ×

$-1 < x \leq 6$ (4)

④ $5 + 2x = 3x - 1$

$x = 6$

$5 + 2x = -3x + 1$

$5x = -4$

$x = -\frac{4}{5}$

Test

$x = 6$ $|17| = 17$ ✓

$x = -\frac{4}{5}$ $3^2/5 = -3^2/5$ ×

Solution $x = 6$ (3)

(d) $(a+b)(a^2+ab+b^2) + (a+b)$

$= (a+b)(a^2+ab+b^2+1)$ (2)

(e) (i) $f(3) = 1$

$g(2) = 1 + 2 + 3$

$= 6$

$f(3) - g(2) = 1 - 6$

$= -5$ (3)

(ii) $g(a-1) = (a-1)^2 + 2(a-1) + 3$

$= a^2 - 2a + 1 + 2a - 2 + 3$

$= a^2 + 2$

Question 2

a) $26 \times 26 \times 10 \times 10 \times 10 = 676{,}000$ (2)

b) 1st 2nd 3rd 4th

(i) $9 \times 9 \times 8 \times 7 = 4536$ (2)

(ii) $9 \times 9 \times 8 \times 7 = 2016$ (2)

(iii) 4th 1st 2nd 3rd

$5 \times 8 \times 8 \times 7 = 2240$ (2)

c) (i) $^{7}C_{6} = 7$ (2)

(ii) $^{7}C_{3} \times {}^{8}C_{3} = 1960$ (2)

(iii) $^{14}C_{6} = 3003$ (2)

(iv) 4 girls 2 boys $^{8}C_{4} \times {}^{7}C_{2} = 1470$

5 girls 1 boy $^{8}C_{5} \times {}^{7}C_{1} = 392$

6 girls $^{8}C_{6} = 28$

Total = 1890 (4)

(v) Two girls together

$^{13}C_{4} = 715$

Not together

$^{15}C_{6} - {}^{13}C_{4} = 4290$ (3)

(d) (i) $6! = 720$ (1)

(ii) m c m

4 w w 2

3 w w 1

$2 \times 4 \times 3 \times 2 = 48$ (2)

(iii) 4 1 3

w c w

4 w w 3

m m

2 1

$4 \times 3 \times 4 \times 3 \times 2 = 12 \times 24$

$= 288$ (2)

(e) (i) $\frac{10!}{2!\,2!\,2!} = 453600$

R O T

(3)

(ii) (II) BRROOHNE

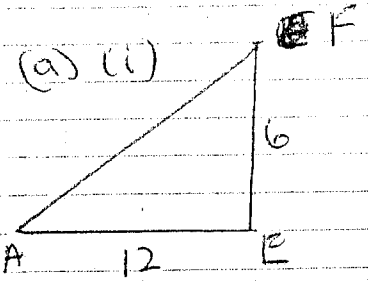
$\frac{9!}{2!\,2!} = \frac{90720}{483600}$

O R

$= \frac{1}{5}$ (2)

QUESTION 3

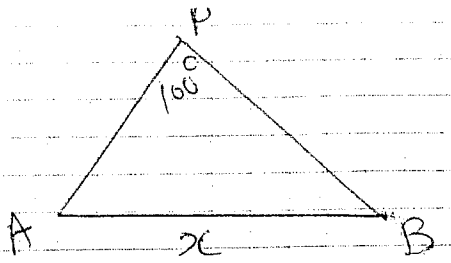
(a) (i)



$AF^2 = 144 + 36$

$= 180$

$\therefore AF = 6\sqrt{5}$ (2)



$x^2 = 50^2 \cot^2 35 + 50^2 \cot^2 50^\circ$

$- 2 \times 50 \cot 35^\circ \times 50 \cot 50^\circ \cos 100^\circ$

$\therefore AB^2 = 50^2 (\cot^2 35 + \cot^2 50 - 2 \cot 35^\circ \cot 50^\circ \cos$

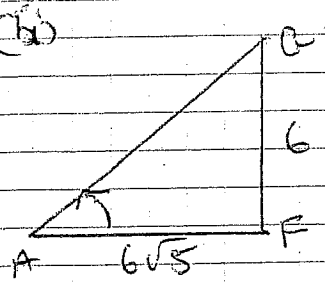
(3)

(ii) ~~AB = ...~~

$AB = 88.88$ correct to nearest m (1)

$AB = 89$

(b)



~~∠FAG~~

$\tan \hat{FAG} = \frac{6}{6\sqrt{5}}$

$\hat{FAG} = 24^\circ 6'$ (2)

(b) (i) $125 - 25 = 100^\circ$ (difference between two bearings)

(2)

(ii) $\frac{50}{AP} = \tan 35^\circ$

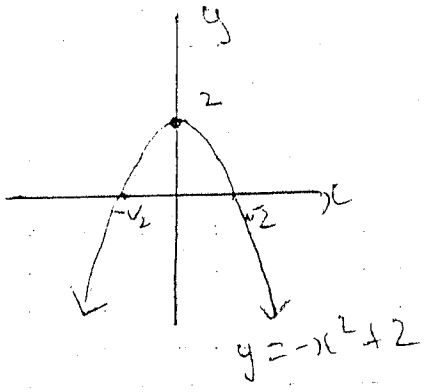
$\therefore AP = \frac{50}{\tan 35^\circ}$

$= 50 \cot 35^\circ$ (2)

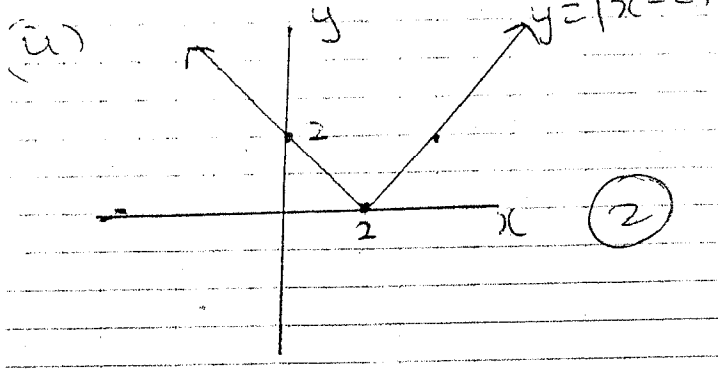
(iii) $BP = 50 \cot 50$

Using cosine rule

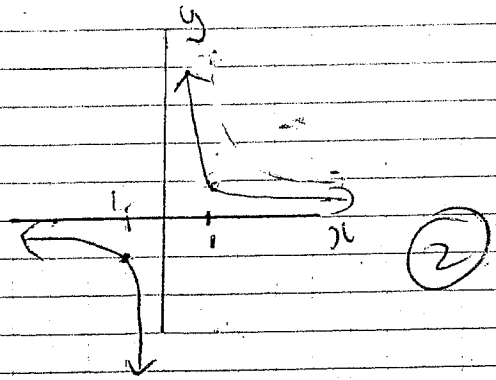
QUESTION 4

(c) 

$y = -x^2 + 2$ (2)

(ii) 

(2)

(iii) 

(2)

(b) (i) domain all real x

range $y \geq 0$ (2)

(ii) ~~odd~~

domain all real, $x \neq 2$

range all real, $y \neq 0$ (2)

(iii) domain all real x

range $y \geq 0$. (2)

(c) Function since satisfies vertical line test. (2)

(d) even since

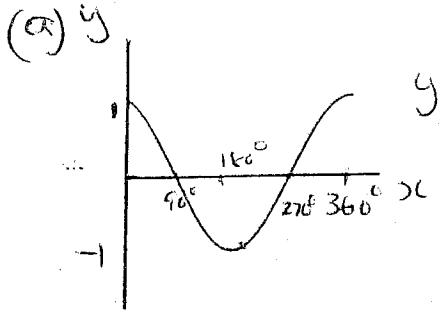
$f(-x) = (-x)^2 + 8$

$= x^2 + 8$

$= f(x)$ (2)

QUESTION 5

(a)



$y = \cos x$.

(3)

(b)(c) $4\cos x + 4(1-\cos^2 x) = 5$

$4\cos^2 x - 4\cos x + 1 = 0$

$(2\cos x - 1)^2 = 0$

$\cos x = \frac{1}{2}$

$x = 60°, 300°$ (4)

(d) $\cos 2\theta = -\frac{\sqrt{3}}{2}$

$2\theta = 150°, 210°, 510°, 570°$

$\theta = 75°, 105°, 255°, 285°$ (3)

(c) $\dfrac{\cos A - \frac{\sin A}{\cos A}\sin A}{\cos A + \frac{\sin A}{\cos A}\sin A}$

$= \dfrac{\cos^2 A - \sin^2 A}{\cos^2 A + \sin^2 A}$.

$= \cos^2 A - \sin^2 A$

$= 1 - \sin^2 A - \sin^2 A$

$= 1 - 2\sin^2 A$ (4)

(d) $\frac{1}{a} = \sqrt{5} - 2$

$\therefore a = \frac{1}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2}$

$= \frac{\sqrt{5}+2}{5-4}$

$LHS = \sqrt{5} + 2$

$RHS = 4 + \frac{1}{a} = 4 + \sqrt{5} - 2$

$= \sqrt{5} + 2$.

$\therefore LHS = RHS$

(3)