



Girraween High School
Mathematics Extension 1
Year 11 Half Yearly Examination
May 2007

Time allowed – 90 minutes

- INSTRUCTIONS:
1. Attempt all questions.
 2. Write your answers on your own paper.
 3. On each page, clearly write:
 - the question being answered
 - your name
 - your Mathematics teacher's name.
 4. Start each question on a new page.
 5. All necessary working must be shown.
 6. Marks will be deducted for careless and badly arranged work.

QUESTION 1 (15 Marks)

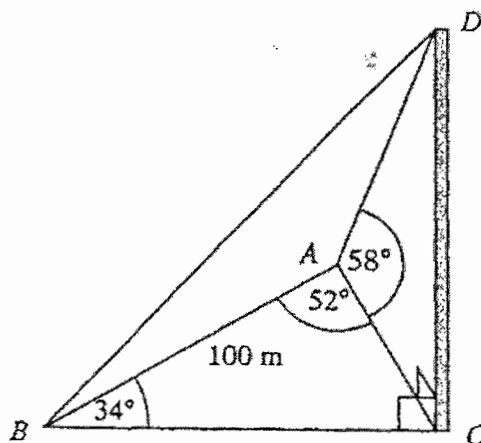
- (a) Solve $|2x+1| \geq 4$ 3
- (b) Factorise $2m^3 - 128$ 2
- (c) Simplify $\frac{1}{x^2-4} - \frac{1}{x^2+3x+2}$ 4
- (d) Solve $\frac{5}{x+2} \leq 1$ 3
- (e) Solve $\frac{2x+1}{3x-2} > 2$ 3

QUESTION 2 (22 Marks) *Start on a new page.*

- (a) Simplify $\tan A \sin A + \cos A$ 4
- (b) Solve for $0^\circ \leq x \leq 360^\circ$:
- (i) $\cos 2\theta = -\frac{\sqrt{3}}{2}$ 3
- (ii) $\tan \theta + 2 \cot \theta = 3$ 4
- (iii) $\sec^2 x + \tan x - 7 = 0$ 4
- (iv) $2 \cos^2 \theta = \cos \theta$ 3
- (c) Prove $(1 + \cot^2 \theta)(1 - \cos^2 \theta) = 1$ 4

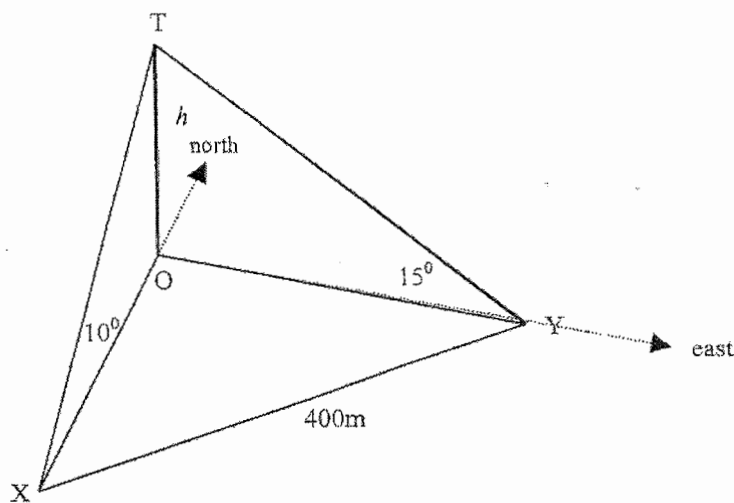
QUESTION 3 (15 Marks) *Start on a new page.*

- (a) A pole CD is seen from the two points A and B .
The angle of elevation from A to the top of the pole is 58° .



If $\angle CAB = 52^\circ$ and $\angle ABC = 34^\circ$, and A and B are 100 metres apart, find:

- (i) How far A is from the foot of the pole, to the nearest metre. 3
 - (ii) The height of the pole, correct to 1 decimal place. 3
- (b) A surveyor stands at a point X and observes a tower due north.
The angle of elevation to the top of the tower is 10° .
The surveyor then walks 400m to a position Y which is due east of the tower.
The angle of elevation from Y to the top of the tower is 15° .



- (i) Write an expression for OY in terms of h . 2
- (ii) Calculate the height of the tower, h , correct to the nearest metre. 4
- (iii) Find the bearing of Y from X . 3

QUESTION 4 (28 Marks) *Start on a new page.*

- (a) A security lock has 7 buttons labelled as shown.

<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>
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Each person using the lock is given a 3 letter code.

- (i) How many different codes are possible if letters can be repeated and their order is important? 2
- (ii) How many different codes are possible if letters cannot be repeated and their order is important? 2
- (iii) Now suppose that the lock operates by holding 3 buttons down together, so that the order is NOT important. How many different codes are possible? 2

- (b) Consider the letters of *ALGEBRAIC*.

- (i) How many distinct arrangements of the letters are possible? 2
- (ii) How many distinct arrangements are possible if the two letter *A*'s must be placed so one is at the beginning and one at the end? 2
- (iii) How many distinct arrangements are possible if the two letter *A*'s must be placed together? 2

- (c) An 8 person committee is to be formed from a group of 10 women and 15 men.

In how many ways can the committee be chosen if:

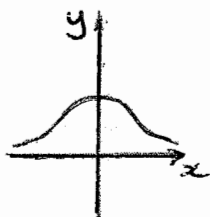
- (i) The committee must contain 4 men and 4 women? 3
- (ii) There must be more women than men? 4
- (iii) There must be at least two women? 4
- (iv) If Jamiel and Leo can't be on the committee together? 2

- (d) In how many ways can 6 men and 4 women be seated around a table if a husband and wife want to sit together? 3

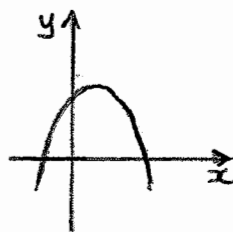
QUESTION 5 (20 Marks) Start on a new page.

(a) State whether the following are functions or not:

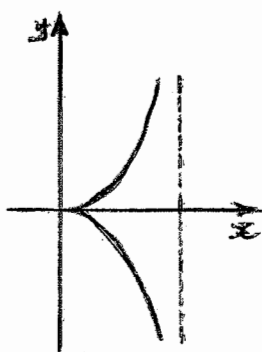
(i)



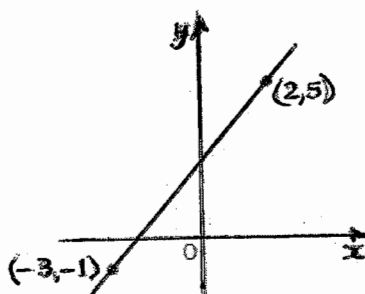
(ii)



(iii)



(iv)



4

(b) State domain and range of:

(i) $y = x^2 - 1$

2

(ii) $y = \frac{1}{x+2}$

2

(iii) $y = \sqrt{x}$

2

(c) If $f(x) = x^2 - 3x + 2$, find the value of:

(i) $f(2)$

2

(ii) $f(a)$

1

(iii) $f(-a)$

1

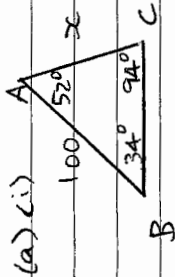
(iv) $f(a) + f(-a)$

2

(d) Find all values of x and y , if $\frac{1}{x} + \frac{3}{y} = 3$ and $\frac{1}{x} + y = 1$.

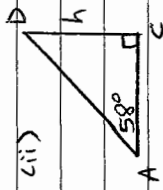
4

QUESTION 3 (15 Marks)



(a) (i) $\frac{x}{\sin 34^\circ} = \frac{100}{\sin 94^\circ}$
 $x = \frac{100 \sin 34^\circ}{\sin 94^\circ}$
 $x = 56.05583955$

$\therefore AC = 56 \text{ m}$ (nearest m) **(3m)**

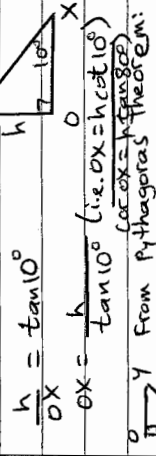


(ii) $\tan 58^\circ = \frac{h}{AC}$
 $\therefore h = AC \times \tan 58^\circ$
 $= 56.05583955 \times \tan 58^\circ$
 $= 89.70809559$
 $h = 89.7 \text{ m}$ (1 decimal place) **(3m)**

Note: $\tan 58^\circ = \frac{h}{56}$ (using rounded answer)
 $h = 56 \times \tan 58^\circ$
 $= 89.61873363$
 $h = 89.6 \text{ m}$

(b) (i) $\frac{h}{OY} = \tan 15^\circ$
 $\therefore OY = \frac{h}{\tan 15^\circ}$ **(2m)**
 (or $OY = h \cot 15^\circ$)
 (or $OY = h \tan 75^\circ$)

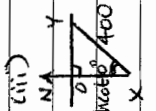
(b) (ii)



$\frac{h}{OX} = \tan 10^\circ$
 $OX = \frac{h}{\tan 10^\circ}$ (i.e. $OX = h \cot 10^\circ$)
 From Pythagoras Theorem:
 $400^2 + OX^2 = x^2$
 $400^2 + \left(\frac{h}{\tan 10^\circ}\right)^2 = x^2$

i. $h^2 \cot^2 10^\circ + h^2 \cot^2 15^\circ = 160000$
 $h^2 (\cot^2 10^\circ + \cot^2 15^\circ) = 160000$
 $\therefore h^2 = \frac{160000}{\cot^2 10^\circ + \cot^2 15^\circ}$
 $= \frac{160000}{46.09164071}$

$h^2 = 3471.345293$
 $h = 58.91812363$
 $\therefore h = 59$ (nearest m) **(4m)**



(iii) $\cos \angle OXY = \frac{OX}{400}$
 $= \frac{h \cot 10^\circ}{400}$
 $\cos \angle OXY = 0.83535208$
 $\angle OXY = 33^\circ 21'$
 (Note: $\angle OXY = 33^\circ 20' 50.52''$)

\therefore bearing of Y from X is **033° (or 033° 21')** **(3m)**

Note: Use rounded value of h:
 $\cos \angle OXY = \frac{59 \times \cot 10^\circ}{400}$
 $\cos \angle OXY = 0.836514068$
 $\angle OXY = 33^\circ 14'$
 bearing **033° (or 33° 14')**

QUESTION 4 (28 Marks)

(a) (i) $7 \times 7 \times 7 = 343$ **(2m)**
 (ii) $7 \times 6 \times 5 = 210$ **(2m)**

(iii) ${}^7C_3 = 35$ **(2m)**

Total ways, more women than men: $138\ 555$ **(4m)**

(c) (ii) Must be at least 2 women:
 Consider complement of this event -
 i.e. consider less than 2 women on
 Committee:

0 women (8 men): ${}^{15}C_8 = 6435$
 1 woman (7 men): ${}^{15}C_1 \times {}^{15}C_7 = 64\ 350$
 Total: less than 2 women $70\ 785$
 \therefore At least 2 women

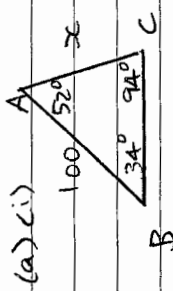
$= (\text{total arrangements}) - (\text{less 2 women})$
 $= 25C_8 - 70\ 785$
 $= 1\ 081\ 575 - 70\ 785$
 $= 1\ 010\ 790$ **(4m)**

(iv) consider complement event:
 If Jamiel & Leo on committee together:
 ${}^{23}C_6 = 100\ 947$ (ways filling other 6 places)
 So Not on committee together
 $= 1\ 081\ 575 - 100\ 947$
 $= 980\ 628$ **(2m)**

(c) 8 people committee
 group 10 women, 15 men
 Committee: 4 men, 4 women
 ${}^{15}C_4 \times {}^{10}C_4 =$
 (men) (women) $286\ 650$ **(3m)**

Ways husband, wife together
 $= 8! \times 2$
 \uparrow
 8 people husband wife
 $= 80\ 640$ **(3m)**

QUESTION 3 (15 Marks)



(a)(i) $\frac{h}{OX} = \tan 10^\circ$

$OX = \frac{h}{\tan 10^\circ}$ (i.e. $OX = h \cot 10^\circ$)
From Pythagoras Theorem:

$400^2 = OX^2 + h^2 = 400^2 + h^2 \cot^2 10^\circ$

$h^2 (\cot^2 10^\circ + 1) = 160000$

$h^2 (\cot^2 10^\circ + \tan^2 10^\circ) = 160000$

$h^2 = \frac{160000}{\cot^2 10^\circ + \tan^2 10^\circ}$

$h = \frac{160000}{46.09164071}$

$h = 3471.345293$

$h = 58918.2363$

$h = 59$ (nearest m)

$\cos \angle OXY = \frac{OX}{400} = \frac{h \cot 10^\circ}{400}$

$\cos \angle OXY = 0.83535208$

$\angle OXY = 33^\circ 21'$

Note: bearing of Y from X is 033° (or $033^\circ 21'$)

Note: Use rounded value of h:

$\cos \angle OXY = \frac{59 \times \cot 10^\circ}{400}$

$\cos \angle OXY = 0.836514068$

$\angle OXY = 33^\circ 14'$

bearing 033° (or 33.14°)

$\frac{h}{OY} = \tan 15^\circ$

$OY = \frac{h}{\tan 15^\circ}$

$OY = \frac{56}{\tan 15^\circ} = 207.7$

(b)(ii)



$OX = \frac{h}{\tan 10^\circ}$

$1000^2 = OX^2 + 400^2 = 400^2 + h^2 \cot^2 10^\circ$

$h^2 (\cot^2 10^\circ + 1) = 1600000$

$h^2 (\cot^2 10^\circ + \tan^2 10^\circ) = 1600000$

$h^2 = \frac{1600000}{\cot^2 10^\circ + \tan^2 10^\circ}$

$h = \frac{1600000}{46.09164071}$

$h = 3471.345293$

$h = 58918.2363$

$h = 59$ (nearest m)

$\cos \angle OXY = \frac{OX}{400} = \frac{h \cot 10^\circ}{400}$

$\cos \angle OXY = 0.83535208$

$\angle OXY = 33^\circ 21'$

Note: bearing of Y from X is 033° (or $033^\circ 21'$)

Note: Use rounded value of h:

$\cos \angle OXY = \frac{59 \times \cot 10^\circ}{400}$

$\cos \angle OXY = 0.836514068$

$\angle OXY = 33^\circ 14'$

bearing 033° (or 33.14°)

$\frac{h}{OY} = \tan 15^\circ$

$OY = \frac{h}{\tan 15^\circ}$

$OY = \frac{56}{\tan 15^\circ} = 207.7$

QUESTION 4 (28 Marks)

(a)(i) $7 \times 7 \times 7 = 343$ (2m)

(ii) $7 \times 6 \times 5 = 210$ (2m)

(iii) ${}^7C_3 = 35$ (2m)

In other ways, more women than men: 138555 (4m)

(iii) Must be at least 2 women:

Consider complement of this event - i.e. consider less than 2 women on committee:

0 women (8 men): ${}^{15}C_8 = 6435$

1 woman (7 men): ${}^{15}C_1 \times {}^{15}C_7 = 64350$

Total: less than 2 women 70785

\therefore At least 2 women $= 1081575 - 70785 = 1010790$ (4m)

(iv) consider complement event: If James & Leo on committee together: ${}^{23}C_6 = 100947$ (ways filling other 6 places)

So not on committee together $= 1081575 - 100947 = 980628$ (2m)

(a) $(H, W) \times \dots$

$\times \times \times \times$

$\times \times \times \times$

$\times \times \times \times$

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$\times \times \times \times$

$\times \times \times \times$

$\times \times \times \times$

(b)(i) ALGEBRAIC

Note: 2 A's

Distinct arrangements $= \frac{9!}{2!} = 181440$ (2m)

(ii) 7A_7 (7 letters)

just arrange 7 ways

Arrangements $= 7! = 5040$ (2m)

(iii) 8A_8 (8 objects)

Arrangements $= 8! = 40320$ (2m)

(c) 8 people committee

group 10 women, 15 men

(i) committee: 4 men, 4 women

${}^{15}C_4 \times {}^{10}C_4 = 286650$ (3m)

(men) (women)

$\times \times \times \times$

$\times \times \times \times$

$\times \times \times \times$

$\times \times \times \times$

$\times \times \times \times$

$\times \times \times \times$

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$\times \times \times \times$

$\times \times \times \times$

$\times \times \times \times$

(ii) 8A_8 (8 objects)

Arrangements $= 8! = 40320$ (2m)

(c) 8 people committee

group 10 women, 15 men

(i) committee: 4 men, 4 women

${}^{15}C_4 \times {}^{10}C_4 = 286650$ (3m)

(men) (women)

$\times \times \times \times$

$\times \times \times \times$

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QUESTION 5 (20 Marks)

- (a) (i) Function (1m)
- (ii) Function (1m)
- (iii) Not a function (1m)
- (iv) Function (1m)

(d) $\frac{1}{x} + \frac{3}{y} = 3$ (1)

$\frac{1}{x} + y = 1$ (2)

From (2): $\frac{1}{x} = 1 - y$ (3)

Substitute (3) into (1):

$(1-y) + \frac{3}{y} = 3$

$y - y^2 + 3 = 3y$

$y^2 + 2y - 3 = 0$

$(y+3)(y-1) = 0$

$y = -3, 1$

when $y = -3$: subst in (2):

$\frac{1}{x} - 3 = 1$

$\frac{1}{x} = 4$

$x = \frac{1}{4}$

when $y = 1$: subst in (2):

$\frac{1}{x} + 1 = 1$

$\frac{1}{x} = 0$

this has no solution.

\therefore only solution is

$x = \frac{1}{4}, y = -3$ (4m)

(b) (i) $y = x^2 - 1$

Domain: all real x

Range: all reals, $y \geq -1$ (2m)

(ii) $y = \frac{1}{x+2}$

Domain: all reals, $x \neq -2$

Range: all reals, $y \neq 0$ (2m)

(iii) $y = \sqrt{x}$

Domain: all reals $x \geq 0$

Range: all reals $y \geq 0$ (2m)

(c) (i) $f(x) = x^2 - 3x + 2$

$f(2) = 2^2 - 6 + 2$

$= 0$ (2m)

(ii) $f(a) = a^2 - 3a + 2$ (1m)

(iii) $f(-a) = (-a)^2 - 3(-a) + 2$

$= a^2 + 3a + 2$ (1m)

(iv) $f(a) + f(-a)$

$= a^2 - 3a + 2 + a^2 + 3a + 2$

$= 2a^2 + 4$ (2m)