MATHEMATICS

YEAR 11   May, 2004

TEST 2   TIME: 90 minutes

INSTRUCTIONS: 1. Attempt all questions.
                 2. Write your answers on your own paper.
                 3. All necessary working must be shown.
                 4. Marks will be deducted for careless or badly arranged work.
                 5. Questions are NOT of equal value.
                 6. Start each question on a new page.
                 7. Total marks – 120

Question 1   (16 marks)

(a) Evaluate $(3.2)^{25} \div 0.015$ expressing your answer in scientific notation correct to 3 significant figures.   /2

(b) Evaluate $\sqrt[3]{\dfrac{9.4 + 3.8}{6.5 - 1.2 \times 3.7}}$ correct to 2 decimal places.   /2

(c) If $\sqrt{5x} = 5\sqrt{2}$,   find x.   /2

(d) Write 45g as a percentage of 2.5kg.   /1

(e) Expand and simplify:

    (i) $4ab(a - 2b) - 2a^2(b - 3a)$ /2

    (ii) $(y^2 - 2)(y^2 + 2)$ /1

(f) Simplify, giving exact answers:

    (i) $\sqrt{48} + \sqrt{27}$ /2

    (ii) $(\sqrt{5} + 2\sqrt{3})^2$ /2

(g) Simplify $\dfrac{3}{\sqrt{5} - 1}$ by rationalising the denominator.   /2

Question 2 (24 marks)

(a) Factorise:

    (i) $8 - 18x^2$ (ii) $x^2 - 7x + 12$ (iii) $3x^2 - 20x - 7$ /6

(b) Factorise and simplify:

    (i) $\dfrac{a^2 - 16}{a + 4}$ (ii) $\dfrac{m^2 - 10m + 25}{m^2 - 2m - 15}$ (iii) $\dfrac{9 - t^2}{t - 3}$ /7

(c) Simplify:

    (i) $\dfrac{4}{y - 2} \div \dfrac{6}{y - 2}$ (ii) $\dfrac{4m - 12}{m^2 - 9} \times \dfrac{3m + 9}{6m + 12}$ /5

(d) Simplify:

    (i) $\dfrac{5 - u}{3} - \dfrac{u + 2}{2}$ (ii) $\dfrac{5}{x - 2} - \dfrac{4}{x + 2}$ /6

Question 3 (25 marks)

(a) Solve the equation:

    (i) $3(x - 5) = 3 - 2(x - 1)$ (ii) $\dfrac{2x - 1}{3} = 1 - \dfrac{x - 2}{5}$ /6

(b) Solve the quadratic equation $6x^2 = 4x + 2$ (leaving answer in simplified surd form if necessary). /3

(c) The equation $x^2 - ax + 20 = 0$ has one solution $x = 4$.
    Find a and the other solution.   /3

(d) Solve the simultaneous equations: $2x + y = 8$
                                      $xy = 6$

(e) Solve $2 - x \le \dfrac{3x}{4}$ and graph the solution on the number line.   /3

(f) Solve the following:

    (i) $|3y + 2| = |6 - 4y|$ (ii) $|2m - 5| > 9$ /6

Question 4   (13 marks)

(a) Find, correct to 3 decimal places:

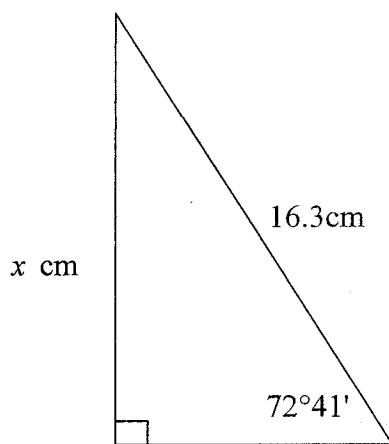(i) $\sin 42^{0}05'$ (ii) $\cos 116^{0}28'$ /2

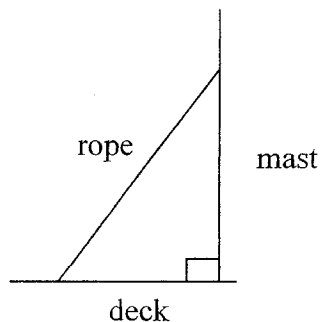(b) If $\tan \theta = \dfrac{6.1}{8.5}$, find θ to the nearest minute.   /2
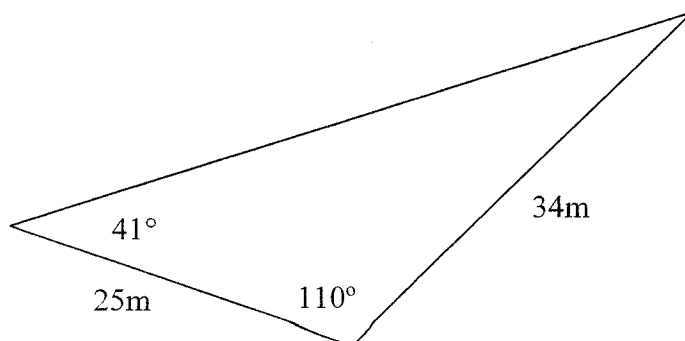
(c) Find the value of x correct to 1 decimal place.

16.3cm

x cm

72°41'

/2

(d) A wire rope is attached to the mast of a boat` 3.2m above the deck, and to the deck, making an angle of elevation of $72^{\circ} 10'$.   How long is the rope?

rope   mast

deck

/3

(e) A triangular block of land is to be made into a park and planted with grass. What is the area to be grassed, to the nearest m² ?

34m

41°

25m   110°

/3

Question 5   (23 marks)

(a)   Find the exact value of:

   (i) $\sin 210°$ (ii) $\cos(-135°)$ (iii) $\cot 300°$ /6

(b)   Find the value(s) of θ (to the nearest degree) for $0° < \theta < 360°$ for which:

   (i) $\cos\theta = 0.917$ (ii) $\sin\theta = -0.158$ (iii) $\tan\theta = 0.674$ and $\sin\theta$ is negative   /6

(c)   Give the exact value, in simplest form:

   (i) $\cot 30° + \cot 60°$ (ii) $\dfrac{\sin 45^0 + \sin 30^0}{\cos 45^0 - \cos 60^0}$ /8

(d)   If $\sin\theta = \dfrac{-5}{13}$ and $\cos\theta < 0$, find the exact value of $\tan\theta$.   /3
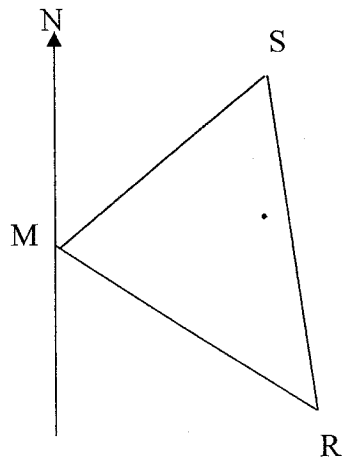

Question 6   (20 marks)

(a)   Solve the following for $0° \le \theta \le 360°$

   (I) $2\cos\theta = \sqrt{3}$ (ii) $\operatorname{cosec}^2\theta - 2 = 0$ /7

(b)   Simplify $\cos^2\theta(1 + \cot^2\theta)$.   /2

(c)   Prove $\dfrac{\operatorname{cosec}\theta}{\cot\theta} = \sec\theta$.   /2

(d)
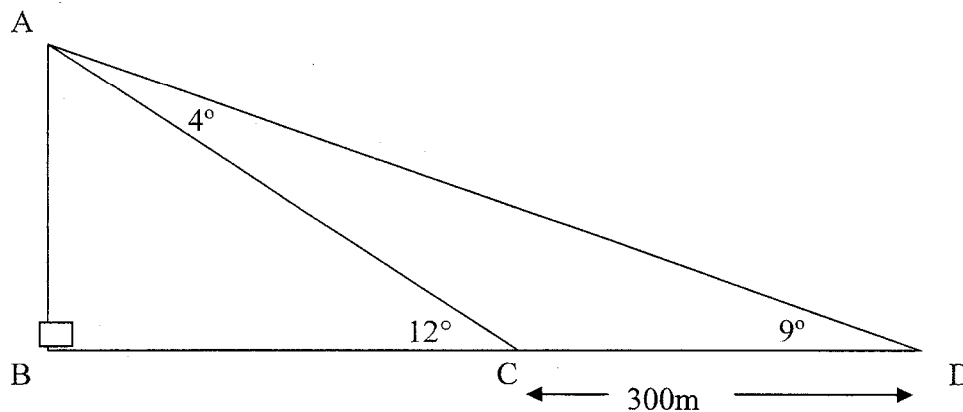
   (i)   Use the sine rule to calculate AC to the nearest metre.   /2

   (ii)   Hence, find BC to the nearest metre.   /2

(e)   A boat (S) is sinking 2.2km out to sea from a marina (M). Its bearing is 041° from the marina. A rescue boat (R) is at bearing 123° from the marina and 2.9km out to sea.

(i)   Copy the diagram and indicate the given information.   /1

(ii)   Show $\angle SMR = 82°$.   /1

(iii)   Use the cosine rule to find the distance RS of the rescue boat from the sinking boat.   /3

Question 1 $\boxed{/18}$

(a) $=2.13 \times 10^{-7} = \boxed{2.13 \times 10^{-7}}$ ①
    (3 sig fig) — ① $\frac{1}{2}$

(b) $\sqrt[3]{\dfrac{9.4+3.5}{\sqrt{65-1.2\times3.7}}} = \sqrt[3]{\dfrac{}{\sqrt{6.4077\ldots}}}$

    $= 1.85788\ldots$ ①
    $\boxed{1.86}$ to 2 dec. pl. $\frac{1}{2}$

(c) $\sqrt{5x} = 5\sqrt{2}$
    $= \sqrt{50}$
    $\therefore 5x = 50$
    $\boxed{x = 10}$ $\frac{1}{2}$

(d) $\dfrac{4g}{2.5kg} = \boxed{0.16\%}$ ①

(d) (i) $4a^2b - 8ab^2 - 2a^2b + 6a^3$
    $= \boxed{6a^3 + 2a^2b - 8ab^2}$ $\frac{1}{2}$

(ii) $\boxed{y^4 - 4}$ ①

(e) (i) $\sqrt{48} + \sqrt{27} = \sqrt{16}\times\sqrt{3} + \sqrt{9}\times\sqrt{3}$
    $= 4\sqrt{3} + 3\sqrt{3}$
    $= \boxed{7\sqrt{3}}$ $\frac{1}{2}$

(ii) $(\sqrt{5})^2 + 2\times2\sqrt{5} + (2\sqrt{3})^2$
    $= 5 + 4\sqrt{15} + 4\times3$
    $= \boxed{17 + 4\sqrt{15}}$ $\frac{1}{2}$

(f) $\dfrac{3}{\sqrt{5}-1} \times \dfrac{\sqrt{5}+1}{\sqrt{5}+1} = \dfrac{3(\sqrt{5}+1)}{5-1}$
    $= \boxed{\dfrac{3\sqrt{5}+3}{4}}$ $\frac{1}{2}$

Question 2 $\boxed{/24}$

/6 (a) (i) $8 - 18x^2 = 2(4 - 9x^2)$
    $= \boxed{2(2+3x)(2-3x)}$ $\frac{1}{2}$

(ii) $x^2 - 7x + 12 = \boxed{(x-3)(x-4)}$ $\frac{1}{2}$

(iii) $3x^2 - 20x - 7 = \boxed{(3x+1)(x-7)}$

    $\begin{array}{cc} 3x & +1 \\ x & -7 \end{array}$ or $\begin{array}{l} 3x \cdot -7 = -21 \\ \rightarrow 3x^2 - 21x + x - 7 \\ = 3x(x-7) + 1(x-7) \\ = (3x+1)(x-7) \end{array}$ $\begin{array}{c} \times -21 \\ \hline +1 \\ \hline -20 + \end{array}$ $\frac{1}{2}$

/7 (b) (i) $\dfrac{a^2-16}{a+4} = \dfrac{(a+4)(a-4)}{(a+4)}$
    $= \boxed{a-4}$ $\frac{1}{2}$

(ii) $\dfrac{m^2-10m+25}{m^2-2m-15} = \dfrac{(m-5)^2}{(m-5)(m+3)}$
    $= \boxed{\dfrac{m-5}{m+3}}$ $\frac{1}{3}$

(iii) $\dfrac{9-t^2}{t-3} = \dfrac{(3-t)(3+t)}{(t-3)}$
    $= \dfrac{-(t-3)(3+t)}{(t-3)}$
    $= -(3+t)$
    $= \boxed{-t-3}$ or $-(t+3)$ $\frac{1}{2}$

/5 (c) (i) $\dfrac{4}{y-2} \div \dfrac{6}{y-2} = \dfrac{\cancel{4}^2}{\cancel{y-2}} \times \dfrac{\cancel{y-2}^1}{\cancel{6}_3}$
    $= \dfrac{2}{3}$ $\frac{1}{2}$

(c) (ii) $\dfrac{4m-12}{m^2-9} \times \dfrac{3m+9}{6m+12} = \dfrac{\cancel{4}(m-3)}{\cancel{(m+3)}\cancel{(m-3)}} \times \dfrac{\cancel{3}(m+3)}{\cancel{6}(m+2)}$

$= \dfrac{2}{m+2}$

$\frac{1}{6}$ (d) (i) $\dfrac{5-u}{3} - \dfrac{u+2}{2} = \dfrac{2(5-u)-3(u+2)}{6}$

$= \dfrac{10-2u-3u-6}{6}$

$= \dfrac{4-5u}{6}$ $\frac{1}{3}$

(ii) $\dfrac{5(x+2)-4(x-2)}{(x-2)(x+2)} = \dfrac{5x+10-4x+8}{x^2-4}$

$= \dfrac{x+18}{x^2-4}$ $\frac{1}{3}$

Question 3 $\boxed{/25}$

$\frac{1}{6}$ (a) (i) $3(x-5) = 3 - 2(x-1)$

$3x-15 = 3-2x+2$

$= 5-2x$

$5x = 20$

$\boxed{x=4}$ $\frac{1}{3}$

(ii) $\dfrac{2x-1}{3} = 1 - \dfrac{x-2}{5}$

$10x-5 = 15-3x+6$

$13x = \cancel{26} 26$

$\boxed{x=2}$ $\frac{1}{3}$

$\frac{1}{3}$ (b)

$6x^2 = 4x+2$

$6x^2-4x-2=0$

$3x^2-2x-1=0$

$(3x+1)(x-1)=0$

$\boxed{x=-\frac{1}{3} \text{ or } 1}$

or $x = \dfrac{4 \pm \sqrt{16+48}}{12}$

$= \dfrac{4 \pm \sqrt{64}}{12}$

$= \dfrac{4 \pm 8}{12}$

$= \dfrac{12}{12} \text{ or } \dfrac{-4}{12}$

$= 1 \text{ or } -\frac{1}{3}$ $\frac{1}{3}$

$\frac{1}{3}$ (c) $x^2 - ax + 20 = 0$

if $x=4$, $16-4a+20=0$

$36-4a=0$

$\boxed{a=9}$ $\diagup 1$

$\therefore x^2-9x+20=0$

$(x-4)(x-5)=0$

$\therefore$ other sol^n is $\boxed{x=5}$ $\frac{1}{2}$

4   (a)

$$2x + y = 8 \quad ①$$
$$xy = 6 \quad ②$$

From ① $y = 8 - 2x \quad ③$
Subst. for y in ②:

$$x(8 - 2x) = 6$$
$$8x - 2x^2 = 6$$
$$2x^2 - 8x + 6 = 0$$
$$x^2 - 4x + 3 = 0$$
$$(x-1)(x-3) = 0$$
$$x = 1, 3$$

Subst. for x in ③:

$$y = 8-2 \text{ or } 8-6$$
$$= 6 \text{ or } 2$$

$\therefore$ Sol^n $\boxed{x=1, y=6 \text{ or } x=3, y=2}$ 4

/3   (c) $2 - x \leq \dfrac{3x}{4}$

$\therefore \; 8 - 4x \leq 3x$
$8 \leq 7x$
$x \geq \dfrac{8}{7}$
ie $x \geq 1\frac{1}{7}$

/6   (f)(i) $|3y + 2| = |6 - 4y|$

Either $3y + 2 = 6 - 4y$ or $3y + 2 = 4y - 6$
 $7y = 4$ $8 = y$
 $y = \dfrac{4}{7}$

$\therefore$ Sol^n $\boxed{y = \dfrac{4}{7} \text{ or } 8}$ /3

(ii) $|2m - 5| > 9$

$2m - 5 > 9$ or $-(2m - 5) > 9$
$2m > 14$ $2m - 5 < -9$
$m > 7$ $2m < -4$
 $m < -2$

$\therefore$ Sol^n $\boxed{m < -2 \text{ or } m > 7}$ /3

Question 4   [/13]

/2 (a) (i) $\sin 42°05' = 0.670210...$
                      $= \boxed{0.670}$ to 3 dec. pl.   ✓

    (ii) $\cos 116°28' = -0.44567...$
                      $= \boxed{-0.446}$ to 3 dec. pl.   ✓

/2 (b) $\tan\theta = \dfrac{6.1}{8.5}$
              $= 0.717647...$
         $\therefore \theta = 35.665...°$
              $= \boxed{35°40'}$ /2

/2 (c) $\tan 72°41' = \dfrac{x}{16.3}$

         $\therefore x = 16.3 \tan 72°41'$
              $= 16.3 \times 3.20734...$
              $= 52.2797...$
              $= \boxed{52.3}$ to 1 dec. pl.   /2

/3 (d)
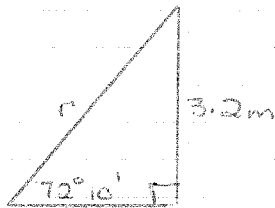
         $\dfrac{3.2}{r} = \sin 72°10'$

         $r = \dfrac{3.2}{\sin 72°10'}$

              $= 3.3615...$
              $\doteq 3.4\,m$
         ∴ Rope is $\boxed{3.4\,m}$ /3

/3 (e) Area $= \frac{1}{2}ab\sin C$

              $= \frac{1}{2} \times 25 \times 34 \sin 110°$ m^2

              $= 399.369$
              $= \boxed{399\,m^2}$ to nearest m^2 /3

/6   (a) (i) $\sin 210° = \sin(180° + 30°)$ (ii) $\cos(-135°) = -\cos 45°$

$= -\sin 30°$ $= \boxed{-\dfrac{1}{\sqrt{2}}}$

$\begin{array}{c|c}S&A\\\hline \boxed{T}&C\end{array}$ $= \boxed{-\dfrac{1}{2}}$ /2 $\begin{array}{c|c}S&A\\\hline \boxed{T}&C\end{array}$ /2

— Possible to just use calculator

(iii) $\cot 300° = -\cot 60°$

$\begin{array}{c|c}S&A\\\hline T&\boxed{C}\end{array}$ $= \boxed{-\dfrac{1}{\sqrt{3}}}$ /2

/6   (b) (i) $\cos\theta = 0.917$ (ii) $\sin\theta = -0.158$

$\theta = 24°$ or $360 - 24°$ $\theta = 180° + 9° = 360° + 9°$

$\theta = \boxed{24° \text{ or } 336°}$ /2 $= \boxed{189° \text{ or } 351°}$ /2

(iii) $\tan\theta = 0.674$

$\theta = 180° + 34°$ (3rd quad only)

$= \boxed{214°}$ /2

(c) (i) $\cot 30° + \cot 60° = \sqrt{3} + \dfrac{1}{\sqrt{3}}$
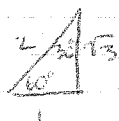
$= \dfrac{3+1}{\sqrt{3}}$ or $\dfrac{\sqrt{3}+\sqrt{3}}{3}$

$= \dfrac{4}{\sqrt{3}}$ $\dfrac{4\sqrt{3}}{3}$

$= \boxed{\dfrac{4\sqrt{3}}{3}}$ /3 ?

/4   (ii) $\dfrac{\sin 45° + \sin 30°}{\cos 45° - \cos 60°} = \dfrac{\dfrac{1}{\sqrt{2}} + \dfrac{1}{2}}{\dfrac{1}{\sqrt{2}} - \dfrac{1}{2}}$

$= \left(\dfrac{\sqrt{2}}{2} + \dfrac{1}{2}\right) \div \left(\dfrac{\sqrt{2}}{2} - \dfrac{1}{2}\right)$

$= \dfrac{\sqrt{2}+1}{2} \times \dfrac{2}{\sqrt{2}-1}$

$= \dfrac{\sqrt{2}+1}{\sqrt{2}-1}$

$= \dfrac{(\sqrt{2}+1)(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)}$

$= \dfrac{2 + 2\sqrt{2} + 1}{2-1}$

$= \boxed{3 + 2\sqrt{2}}$ /4 ?
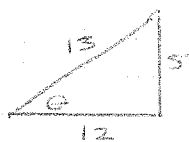
/3   (d) $\sin\theta = \dfrac{-5}{13}$

$\cos\theta < 0$

∴ θ in 3rd quadrant

∴ $\tan\theta > 0$

$\tan\theta = \boxed{\dfrac{5}{12}}$ $13^2 - 5^2 = 144$
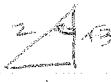
$\sqrt{144} = 12$

/3

        /3

Question 6   1

4 (a) (i) $2\cos\theta = \sqrt{3}$

too much? $\cos\theta = \dfrac{\sqrt{3}}{2}$

$\theta = 30°$ or $360° - 30°$

$= 30°$ or $330°$ /3

(ii) $\csc^2\theta - 2 = 0$

$\csc^2\theta = 2$

$\csc\theta = \pm\sqrt{2}$

$\sin\theta = \pm\dfrac{1}{\sqrt{2}}$

$\theta = \boxed{45°,\ 135°,\ 225°\ \text{or}\ 315°}$

   /4

/2 (b) $\cos^2\theta\,(1 + \cot^2\theta) = \cos^2\theta \times \csc^2\theta$

$= \dfrac{\cos^2\theta}{\sin^2\theta}$

$= \cot^2\theta$ /2

/2 (c)   Pr $\dfrac{\csc\theta}{\cot\theta} = \sec\theta$

LHS $= \dfrac{1}{\sin\theta} \div \dfrac{\cos\theta}{\sin\theta}$

$= \dfrac{1}{\sin\theta} \times \dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{1}{\cos\theta}$

$= \sec\theta$

$=$ RHS   /2

/4 (d)(i) In $\triangle ACD$, $\dfrac{AC}{\sin 9°} = \dfrac{300}{\sin 4°}$

$\therefore AC = \dfrac{300\sin 9°}{\sin 4°}$

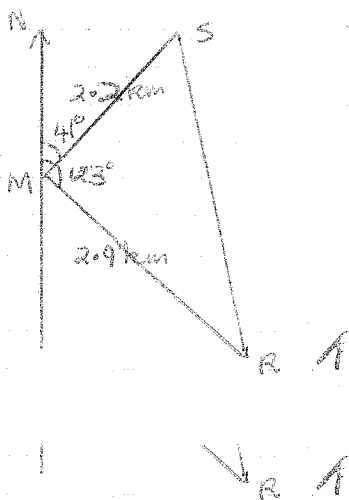$= 672.7\ldots$

$= \boxed{673\ \text{m}}$ to nearest m   /2

(ii) In $\triangle ABC$, $\dfrac{BC}{AC} = \cos 12°$

$\therefore BC = 673\cos 12°$

$= 658.29\ldots$ (or $658.07\ldots$ using all dec.pl.)

$= \boxed{658\ \text{m}}$ to nearest m   /2

/5 (e)(i)

(ii) $\angle SMR = 123° - 41°$

$= 82°$ /1

(iii) $RS^2 = MR^2 + MS^2 - 2\,MR\,MS\cos 82°$

$= (2.9)^2 + (2.2)^2 - 2\times 2.9 \times 2.2\cos 82°$

$= 11.47\ldots$

$\therefore RS = 3.387\ldots$

$= \boxed{3.4\ \text{km}}$ (1 dec pl)   /3

$\ldots RS = 5.38\ldots$

$= \boxed{3.4\ \text{km}}$ (1 dec pl)   /3