

- INSTRUCTIONS:
1. Attempt all questions.
 2. Write your answers on your own paper.
 3. All necessary working must be shown.
 4. Marks will be deducted for careless or badly arranged work.

Question 1 (12 marks)

- a) Find, correct to 3 decimal places. $\frac{3.24^2}{5.73 - 2.84}$ 2
- b) The base length x , of a square pyramid of volume V and perpendicular height h is given by $x = \sqrt{\frac{3V}{h}}$.
Find h , correct to two decimal places if $V = 750$ and $x = 16.32$ 2
- c) Simplify $|3 - 5|^2 - 2| - 2 + 7|$ 2
- d) The speed of light is about 300 000 km/h. Write this speed in scientific notation. 1
- e) Solve $\frac{2x}{x-5} = \frac{3}{5}$ 3
- f) Solve $-1 \leq 2x + 1 \leq 5$ 2

Question 2 (18 marks)

- a) Factorise and simplify:
- (i) $\frac{x-5}{5-x}$ (ii) $\frac{x^2-9}{x-3}$ (iii) $\frac{2(x-3)-4}{2}$ 5
- b) Expand and simplify $(x-1)(x^2-2x-1)$ 2
- c) Factorise fully $2x^2 - 50y^4$ 2
- d) Find integers a and b such that $\frac{2}{2-\sqrt{3}} = a + \sqrt{b}$. 3
- e) Simplify, giving exact answers:
- (i) $\sqrt{45} + \sqrt{80}$ (ii) $(2\sqrt{11}-5)^2$ 4
- f) Rationalise the denominator. $\frac{\sqrt{5}}{2\sqrt{2}-\sqrt{3}}$ 2

Question 3 (21 marks)

- a) Simplify $\frac{4}{t-1} + \frac{t}{t^2-1}$ 2
- b) Factorise (i) $6x^2 - 29x + 28$ (ii) $x^4 - 16$ 4
- c) Solve $\left| \frac{x+1}{3} \right| = 5$ 3
- d) Solve, giving the answers correct to 2 decimal places: $2x^2 - 4x - 3 = 0$ 3
- e) Solve: (i) $|2x - 1| = |x + 5|$ 3
- (ii) $(x - 2)(x - 3) = 12$ 3
- (iii) $x^2 - y^2 = 4$ 3
 $x + y = 1$

Question 4 (11 marks)

- a) Find, correct to 3 decimal places: (i) $\cos 65^{\circ}16'$ (ii) $\tan 135^{\circ}6'$ 2
- b) Find θ correct to the nearest minute: (i) $\tan \theta = 1.7$ (ii) $\cos \theta = \frac{1.6}{2.8}$ 2
- c) Find the value of x , correct to 2 decimal places. 2
- 12 cm

$12^{\circ}10'$

x cm
- d) A ship leaves port and sails on a bearing of 120° for 50 km. 1
- (i) Draw a sketch to show this information. 2
- (ii) How far east of the port is it at this time, correct to the nearest km? 2
- e) Sketch $y = \tan x$ for $0^{\circ} \leq x \leq 360^{\circ}$ 2

Question 5 (20 marks)

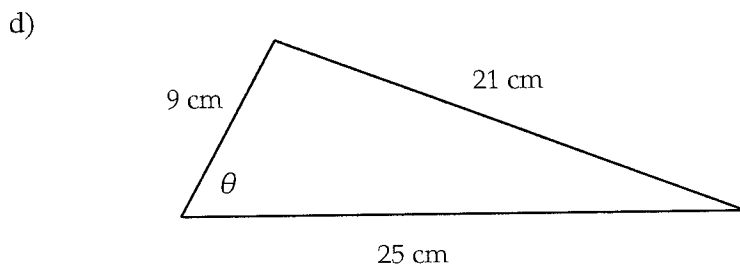
- a) Find the exact value of: 3
 (i) $\sin 150^\circ$ (ii) $\cos 45^\circ$ (iii) $\tan 330^\circ$
- b) Solve for $0^\circ \leq \theta \leq 360^\circ$: 6
 (i) $\sin \theta = \frac{1}{2}$ (ii) $\tan \theta = -\sqrt{3}$
- c) Solve for $0^\circ \leq \theta \leq 360^\circ$: 6
 (i) $4\sin^2 \theta = 3$ (ii) $\cos^2 \theta = 1 - 2\sin^2 \theta$ 3
 (iii) $\tan 2\theta = \frac{1}{\sqrt{3}}$
- d) Simplify fully: $\frac{1}{\cos \theta} - \sin \theta \tan \theta$ 2

Question 6 (12 marks)

- a) Simplify $\sec^2 \theta - \tan^2 \theta$ 2
- b) Prove $(\sin \theta + \cos \theta)^2 + (\sin \theta - \cos \theta)^2 = 2$ 3
- c)



- (i) Use the sine rule to find x , correct to 2 decimal places. 2
 (ii) Find the area of the triangle, correct to the nearest cm^2 . 3

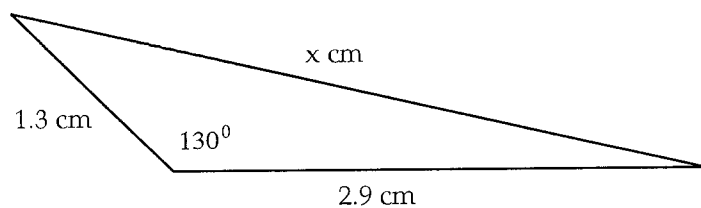


- Use the cosine rule to find the value of θ , correct to the nearest minute. 2

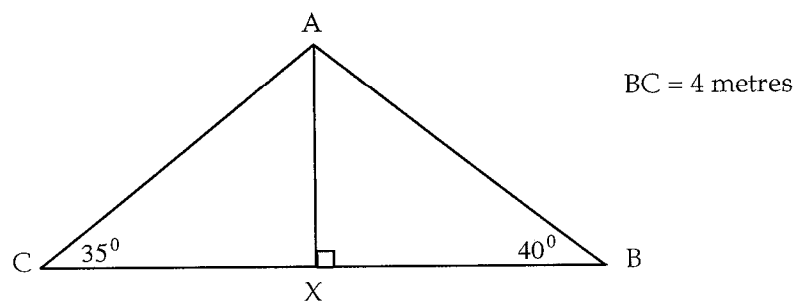
Question 7 (13 marks)

a) Find x , correct to 2 decimal places

2



b)



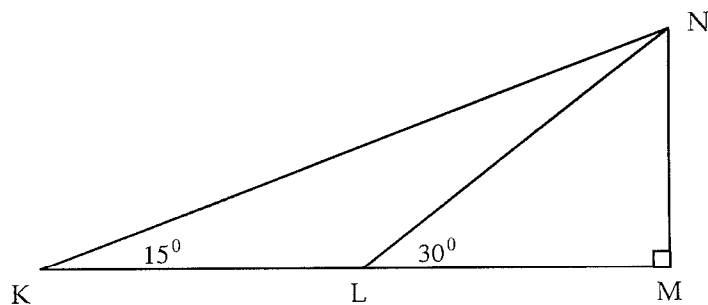
(i) Use the sine rule in $\triangle ABC$ to find AC , correct to 2 decimal places.

3

(ii) Find the length of AX , correct to 2 decimal places.

2

c)



(i) If $KL = 1$ metre, explain why LN also equals 1 metre.

2

(ii) Use the sketch to deduce that: $\tan 15^\circ = 2 - \sqrt{3}$

4

Question 1

a) 3.632

b) $x = \sqrt{\frac{3V}{h}}$

$$16.32 = \sqrt{\frac{3 \times 750}{h}}$$

$$266.3424 = \frac{2250}{h}$$

$$h = \frac{2250}{266.3424}$$

$$h = 8.45$$

c) $|3-5|^2 - 2|-2+7|$
 $= 4 - 2 \times 5$
 $= -6$

d) 3×10^5

e) $\frac{2x}{x-5} = \frac{3}{5}$

$$10x = 3x - 15$$

$$7x = -15$$

$$x = -\frac{15}{7} \text{ or } -2\frac{1}{7}$$

f) $-1 \leq 2x+1 \leq 5$
 $-2 \leq 2x \leq 4$
 $-1 \leq x \leq 2$

Question 2

a) i) -1 ii) $\frac{(x-3)(x+3)}{x-3} = x+3$

iii) $\frac{2(x-3)-4}{2} = x-3-2$
 $= x-5$

b) $x^3 - 2x^2 - x - x^2 + 2x + 1$
 $= x^3 - 3x^2 + x + 1$

c) $2(x^2 - 25y^4) = 2(x-5y^2)(x+5y^2)$

d) $\frac{2}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{4+2\sqrt{3}}{4-3}$

$$= 4 + \sqrt{12}$$

$\therefore a = 4 \quad b = 12$

e) i) $\sqrt{45} + \sqrt{80}$
 $= 3\sqrt{5} + 4\sqrt{5}$
 $= 7\sqrt{5}$

ii) $(2\sqrt{11}-5)^2 = 4 \times 11 - 20\sqrt{11} + 25$
 $= 69 - 20\sqrt{11}$

f) $\frac{\sqrt{5}}{2\sqrt{2}-\sqrt{3}} \times \frac{2\sqrt{2}+\sqrt{3}}{2\sqrt{2}+\sqrt{3}}$
 $= \frac{2\sqrt{10} + \sqrt{15}}{8-3}$
 $= \frac{2\sqrt{10} + \sqrt{15}}{5}$

Question 3

$$\begin{aligned} \text{a) } & \frac{4}{t-1} + \frac{t}{(t-1)(t+1)} \\ & = \frac{4(t+1) + t}{(t-1)(t+1)} \\ & = \frac{5t+4}{t^2-1} \end{aligned}$$

$$\begin{aligned} \text{b) i) } & (3x-4)(2x-7) \\ \text{ii) } & (x^2-4)(x^2+4) \\ & = (x-2)(x+2)(x^2+4) \end{aligned}$$

$$\text{c) } \frac{x+1}{3} = 5 \text{ or } -\left(\frac{x+1}{3}\right) = 5$$

$$\begin{aligned} x+1 &= 15 & \frac{x+1}{3} &= -5 \\ x &= 14 & x+1 &= -15 \\ & & x &= -16 \end{aligned}$$

$$\therefore x = 14 \text{ or } -16$$

$$\text{d) } x = \frac{-(-4) \pm \sqrt{16-4 \times 2 \times (-3)}}{4}$$

$$x = \frac{4 \pm \sqrt{40}}{4}$$

$$x = 2.58 \text{ or } -0.58$$

e)

$$\begin{aligned} \text{i) } & 2x-1 = x+5, \quad -(2x-1) = x+5 \\ & x = 6 & -2x+1 &= x+5 \\ & & -4 &= 3x \\ & & x &= -\frac{4}{3} \end{aligned}$$

$$\begin{aligned} \text{ii) } & (x-2)(x-3) = 12 \\ & x^2-5x+6 = 12 \\ & x^2-5x-6 = 0 \\ & (x-6)(x+1) = 0 \end{aligned}$$

$$\therefore x = -1 \text{ or } 6$$

$$\text{iii) } x^2 - y^2 = 4 \quad y = 1-x$$

$$\begin{aligned} \therefore & x^2 - (1-x)^2 = 4 \\ & x^2 - (1-2x+x^2) = 4 \\ & \cancel{x^2} - 1 + 2x - \cancel{x^2} = 4 \\ & 2x = 5 \\ & x = 5/2 \\ & y = -3/2 \end{aligned}$$

Question 4

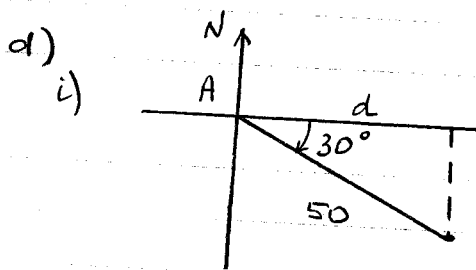
$$\text{a) i) } 0.418 \quad \text{ii) } -0.997$$

$$\text{b) i) } \theta = 59^\circ 32' \quad \text{ii) } 55^\circ 9'$$

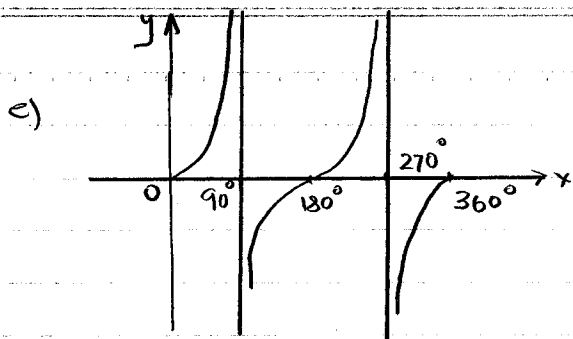
$$\text{c) } \sin 12^\circ 10' = \frac{12}{x}$$

$$x = \frac{12}{\sin 12^\circ 10'}$$

$$x = 56.94 \text{ cm}$$



$$\begin{aligned} \text{ii) } & \cos 30 = \frac{d}{50} \\ \therefore & d = 43 \text{ km} \end{aligned}$$



Question 5

a) i) $\sin 150^\circ = \frac{1}{2}$

ii) $\cos 45^\circ = \frac{1}{\sqrt{2}}$

iii) $\tan 330^\circ = -\frac{1}{\sqrt{3}}$

b) i) $\sin \theta = \frac{1}{2}$

acute angle 30°

quadrants 1 and 2

$\therefore \theta = 30^\circ, 150^\circ$

ii) $\tan \theta = -\sqrt{3}$

acute angle 60°

quadrants 2 and 4

$\therefore \theta = 120^\circ, 300^\circ$

c) i) $4 \sin^2 \theta = 3$

$\sin^2 \theta = \frac{3}{4}$

$\sin \theta = \pm \frac{\sqrt{3}}{2}$

acute angle 60°

quadrants 1, 2, 3 and 4

$\therefore \theta = 60^\circ, 120^\circ, 240^\circ, 300^\circ$

ii) $\cos^2 \theta = 1 - 2 \sin^2 \theta$

$1 - \sin^2 \theta = 1 - 2 \sin^2 \theta$

$\sin^2 \theta = 0$

$\therefore \theta = 0^\circ, 180^\circ, 360^\circ$

ii) $\tan 2\theta = \frac{1}{\sqrt{3}}$

$0 \leq 2\theta \leq 720^\circ$

acute $L: 30^\circ$

$\therefore 2\theta = 30^\circ, 210^\circ, 390^\circ, 570^\circ$

$\therefore \theta = 15^\circ, 105^\circ, 195^\circ, 285^\circ$

d) $\frac{1}{\cos \theta} - \sin \theta \tan \theta$

$= \frac{1}{\cos \theta} - \frac{\sin \theta \cdot \sin \theta}{\cos \theta}$

$= \frac{1 - \sin^2 \theta}{\cos \theta}$

$= \frac{\cos^2 \theta}{\cos \theta}$

$= \cos \theta$

Question 6

a) $\sec^2 \theta - \tan^2 \theta = 1$

b) LHS:

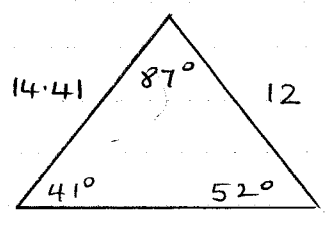
$(\sin \theta + \cos \theta)^2 + (\sin \theta - \cos \theta)^2$
 $= \sin^2 \theta + 2 \sin \theta \cos \theta + \cos^2 \theta$
 $+ \sin^2 \theta - 2 \sin \theta \cos \theta + \cos^2 \theta$
 $= 2(\sin^2 \theta + \cos^2 \theta)$
 $= 2 \text{ RHS}$

c) i) $\frac{x}{\sin 52^\circ} = \frac{12}{\sin 41^\circ}$

$x = \frac{12 \sin 52^\circ}{\sin 41^\circ}$

$x = 14.41$

c ii)



$$\text{Area} = \frac{1}{2} \times 14.41 \times 12 \times \sin 87^\circ$$

$$= 86 \text{ cm}^2$$

d) $\cos \theta = \frac{9^2 + 25^2 - 21^2}{2 \times 9 \times 25}$

$$\cos \theta = 0.58$$

$$\theta = 53^\circ 55'$$

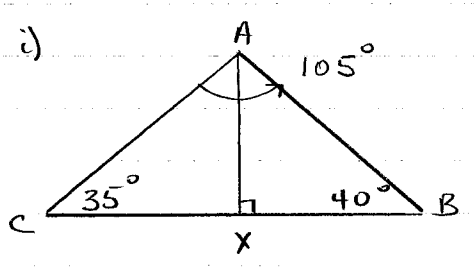
Question 7

a) $x^2 = 1.3^2 + 2.9^2 - 2 \times 1.3 \times 2.9 \times \cos 130^\circ$

$$x^2 = 14.946618 \dots$$

$$x = 3.87 \text{ cm}$$

b) i)



$$\frac{AC}{\sin 40^\circ} = \frac{4}{\sin 105^\circ}$$

$$AC = \frac{4 \sin 40^\circ}{\sin 105^\circ}$$

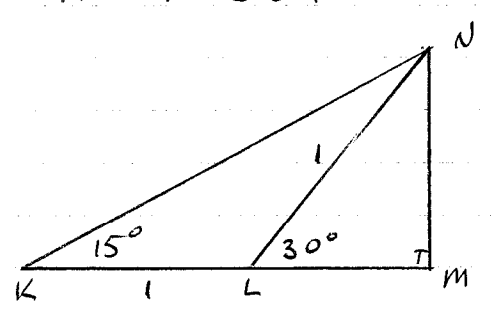
$$AC = 2.66 \text{ m}$$

ii) $\sin 35^\circ = \frac{AX}{2.66}$

$$AX = 2.66 \times \sin 35^\circ$$

$$AX = 1.53 \text{ cm}$$

c)



i) $\angle KNL = 15^\circ$ (sum of interior opposite angles equals exterior angle of $\triangle KLN$)

$\therefore \triangle KLN$ is isosceles
 $\therefore LN = 1 \text{ metre}$

ii) in $\triangle LNM$

- $\cos 30^\circ = \frac{LM}{1}$
- $\frac{\sqrt{3}}{2} = LM$

- $\sin 30^\circ = \frac{MN}{1}$
- $\frac{1}{2} = MN$

in $\triangle KNM$

$$\tan 15^\circ = \frac{MN}{KM}$$

$$= \frac{\frac{1}{2}}{1 + \frac{\sqrt{3}}{2}}$$

$$= \frac{1}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}}$$

$$= \frac{2 - \sqrt{3}}{4 - 3}$$

$$\tan 15^\circ = 2 - \sqrt{3}$$