

# *Permutations*

## **Case 3: Ordered Sets of $n$ Objects, Not All Different**

2 objects

*all different*

**A B**

**B A**

$$2! = 2$$

*2 same*

**A A**

1

*(i.e. some of the objects are the same)*

3 objects

*all different*

**A B C**

**A C B**

**B A C**

**B C A**

**C A B**

**C B A**

$$3! = 6$$

*2 same*

**A A B**

**A B A**

**B A A**

3

*3 same*

**A A A**

1

4 objects

*all different*

<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>	<b>C</b>	<b>B</b>	<b>A</b>	<b>D</b>
<b>A</b>	<b>B</b>	<b>D</b>	<b>C</b>	<b>C</b>	<b>B</b>	<b>D</b>	<b>A</b>
<b>A</b>	<b>C</b>	<b>B</b>	<b>D</b>	<b>C</b>	<b>A</b>	<b>B</b>	<b>D</b>
<b>A</b>	<b>C</b>	<b>D</b>	<b>B</b>	<b>C</b>	<b>A</b>	<b>D</b>	<b>B</b>
<b>A</b>	<b>D</b>	<b>B</b>	<b>C</b>	<b>C</b>	<b>D</b>	<b>B</b>	<b>A</b>
<b>A</b>	<b>D</b>	<b>C</b>	<b>B</b>	<b>C</b>	<b>D</b>	<b>A</b>	<b>B</b>
<b>B</b>	<b>A</b>	<b>C</b>	<b>D</b>	<b>D</b>	<b>A</b>	<b>C</b>	<b>B</b>
<b>B</b>	<b>A</b>	<b>D</b>	<b>C</b>	<b>D</b>	<b>A</b>	<b>B</b>	<b>C</b>
<b>B</b>	<b>C</b>	<b>A</b>	<b>D</b>	<b>D</b>	<b>C</b>	<b>A</b>	<b>B</b>
<b>B</b>	<b>C</b>	<b>D</b>	<b>A</b>	<b>D</b>	<b>C</b>	<b>B</b>	<b>A</b>
<b>B</b>	<b>D</b>	<b>A</b>	<b>C</b>	<b>D</b>	<b>B</b>	<b>A</b>	<b>C</b>
<b>B</b>	<b>D</b>	<b>A</b>	<b>B</b>	<b>D</b>	<b>B</b>	<b>A</b>	<b>B</b>

$$4! = 24$$

*2 same*

<b>A</b>	<b>A</b>	<b>B</b>	<b>C</b>
<b>A</b>	<b>A</b>	<b>C</b>	<b>B</b>
<b>A</b>	<b>B</b>	<b>A</b>	<b>C</b>
<b>A</b>	<b>B</b>	<b>C</b>	<b>A</b>
<b>A</b>	<b>C</b>	<b>A</b>	<b>B</b>
<b>A</b>	<b>C</b>	<b>B</b>	<b>A</b>
<b>B</b>	<b>A</b>	<b>A</b>	<b>C</b>
<b>B</b>	<b>A</b>	<b>C</b>	<b>A</b>
<b>B</b>	<b>C</b>	<b>A</b>	<b>A</b>
<b>C</b>	<b>A</b>	<b>A</b>	<b>B</b>
<b>C</b>	<b>A</b>	<b>B</b>	<b>A</b>
<b>C</b>	<b>B</b>	<b>A</b>	<b>A</b>

$$12$$

*3 same*

<b>A</b>	<b>A</b>	<b>A</b>	<b>B</b>
<b>A</b>	<b>A</b>	<b>B</b>	<b>A</b>
<b>A</b>	<b>B</b>	<b>A</b>	<b>A</b>
<b>B</b>	<b>A</b>	<b>A</b>	<b>A</b>

$$4$$

*4 same*

<b>A</b>	<b>A</b>	<b>A</b>	<b>A</b>
----------	----------	----------	----------

$$1$$

If we arrange  $n$  objects in a line, of which  $x$  are alike, the number of ways we could arrange them are;

$$\text{Number of Arrangements} = \frac{n!}{x!}$$

ways of arranging

$n$  objects

ways of arranging

the like objects

e.g. How many different words can be formed using all of the letters in the word

**CONNAUGHTON ?**

$$\text{Words} = \frac{11!}{2!3!}$$

$$= \underline{3326400}$$

2! for the two O's

3! for the three N's

2001 Extension 1 HSC Q2c)

The letters *A*, *E*, *I*, *O* and *U* are vowels

(i) How many arrangements of the letters in the word **ALGEBRAIC** are possible?

$$\begin{aligned} \text{Words} &= \frac{9!}{2!} \\ &= \underline{181440} \end{aligned}$$

(ii) How many arrangements of the letters in the word **ALGEBRAIC** are possible if the vowels must occupy the 2<sup>nd</sup>, 3<sup>rd</sup>, 5<sup>th</sup>, and 8<sup>th</sup> positions?

$$\begin{aligned} \text{Words} &= \frac{4!}{2!} \times 5! \\ &= \underline{1440} \end{aligned}$$

Number of ways of placing the vowels      Number of ways of placing the consonants

**Exercise 10F; odd**