Trig Integrals with Euler

$$\int \sin^3 x dx = -\frac{1}{8i}\int\left(e^{3ix} - 3e^{ix} + 3e^{-ix} - e^{3ix}\right)dx$$

$$= -\frac{1}{8i}\left[\frac{1}{3i}\left(e^{3ix} + e^{-3ix}\right) - \frac{3}{i}\left(e^{ix} + e^{-ix}\right)\right] + c$$

$$= \frac{1}{12}\cos 3x - \frac{3}{4}\cos x + c$$

$$= \frac{1}{12}\left(4\cos^3 x - 3\cos x\right) - \frac{3}{4}\cos x + c$$

$$= \frac{1}{3}\cos^3 x - \frac{1}{4}\cos x - \frac{3}{4}\cos x + c$$

$$= \frac{1}{3}\cos^3 x - \cos x + c$$

$$\int \sin^4 x dx = \frac{1}{16}\int\left(e^{4ix} - 4e^{2ix} + 6 - 4e^{-2ix} + e^{-4ix}\right)dx$$

$$= \frac{1}{16}\left[\frac{1}{4i}\left(e^{4ix} - e^{-4ix}\right) - \frac{2}{i}\left(e^{2ix} - e^{-2ix}\right) + 6x\right] + c$$

$$= \frac{1}{32}\sin 4x - \frac{1}{4}\sin 2x + \frac{3}{8}x + c$$

$$\int \sin^5 x dx = \frac{1}{32i}\int\left(e^{5ix} - 5e^{3ix} + 10e^{ix} - 10e^{-ix} + 5e^{-3ix} - e^{-5ix}\right)dx$$

$$= \frac{1}{32i}\left[\frac{1}{5i}\left(e^{5ix} + e^{-5ix}\right) - \frac{5}{3i}\left(e^{3ix} + e^{-3ix}\right) + \frac{10}{i}\left(e^{ix} + e^{-ix}\right)\right] + c$$

$$= -\frac{1}{80}\sin 5x + \frac{5}{48}\cos 3x - \frac{5}{8}\cos x + c$$

$$\int \cos^5 x \sin^3 x dx = -\frac{1}{256i}\int \left(e^{ix}+e^{-ix}\right)^5 \left(e^{ix}-e^{-ix}\right)^3 dx$$

$$= -\frac{1}{256i}\int \left(e^{ix}+e^{-ix}\right)^2 \left(e^{2ix}-e^{-2ix}\right)^3 dx$$

$$= -\frac{1}{256i}\int \left(e^{2ix}+2+e^{-2ix}\right)\left(e^{6ix}-3e^{2ix}+3e^{-2ix}-e^{-6ix}\right) dx$$

$$= -\frac{1}{256i}\int (e^{8ix}-3e^{4ix}+3-e^{-4ix}+2e^{6ix}-6e^{2ix}+6e^{-2ix}-2e^{-6ix}$$

$$+e^{4ix}-3+3e^{-4ix}-e^{-8ix})dx$$

$$= -\frac{1}{256i}\left[\frac{1}{8i}\left(e^{8ix}+e^{-8ix}\right)+\frac{1}{3i}\left(e^{6ix}+e^{-6ix}\right)-\frac{1}{2i}\left(e^{4ix}+e^{-4ix}\right)-\frac{3}{i}\left(e^{2ix}+e^{-2ix}\right)\right]$$

$$= \frac{1}{1024}\cos 8x + \frac{1}{384}\cos 6x - \frac{1}{256}\cos 4x - \frac{3}{128}\cos 2x + c$$

$$\int \sin^6 x \cos^3 x dx = -\frac{1}{512}\int \left(e^{ix}+e^{-ix}\right)^3 \left(e^{ix}-e^{-ix}\right)^6 dx$$

$$= -\frac{1}{512}\int \left(e^{ix}-e^{-ix}\right)^3 \left(e^{2ix}-e^{-2ix}\right)^3 dx$$

$$= -\frac{1}{512}\int \left(e^{3ix}-3e^{ix}+3e^{-ix}-e^{-3ix}\right)\left(e^{6ix}-3e^{2ix}+3e^{-2ix}-e^{-6ix}\right)dx$$

$$= -\frac{1}{512}\int \left[\left(e^{9ix}+e^{-9ix}\right)-3\left(e^{7ix}+e^{-7ix}\right)+8\left(e^{3ix}+e^{-3ix}\right)-6\left(e^{ix}+e^{-ix}\right)\right]dx$$

$$= -\frac{1}{512}\left[\frac{1}{9i}\left(e^{9ix}-e^{-9ix}\right)-\frac{3}{7i}\left(e^{7ix}-e^{-7ix}\right)+\frac{8}{3i}\left(e^{3ix}-e^{-3ix}\right)-\frac{6}{i}\left(e^{ix}-e^{-ix}\right)\right]+c$$

$$= -\frac{1}{2304}\sin 9x+\frac{3}{1792}\sin 7x-\frac{1}{96}\sin 3x+\frac{3}{128}\sin x+c$$

$$\int \sin^2 x \cos^2 x dx = -\frac{1}{16}\int \left(e^{ix} - e^{-ix}\right)^2 \left(e^{ix} + e^{-ix}\right)^2 dx$$

$$= -\frac{1}{16}\int \left(e^{2ix} - e^{-2ix}\right)^2 dx$$

$$= -\frac{1}{16}\int \left(e^{4ix} - 2 + e^{-4ix}\right) dx$$

$$= -\frac{1}{16}\left[\frac{1}{4i}\left(e^{4ix} - e^{-4ix}\right) - 2x\right] + c$$

$$= -\frac{1}{32}\sin 4x + \frac{1}{8}x + c$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{e^{ix} - e^{-ix}}{i\left(e^{ix} + e^{-ix}\right)}$$

$$\int \tan^3 x dx = -\frac{1}{i}\int \left(\frac{e^{ix} - e^{-ix}}{e^{ix} + e^{-ix}}\right)^3 dx$$

TOO MUCH WORK INVOLVED COMPARED TO PREVIOUS METHOD

IF USING EULER'S, STICK WITH FUNCTIONS INVOLVING SINX OR COSX