

# Combinations

A combination is a set of objects where the order that they are arranged is not important.

If we arrange objects in a line, and the order is not important then;

**A B** is the same arrangement as **B A**

e.g. 5 objects, arrange 2 of them

<b>A B</b>	<del><b>B A</b></del>	<del><b>C A</b></del>	<del><b>D A</b></del>	<del><b>E A</b></del>
<b>A C</b>	<b>B C</b>	<del><b>C B</b></del>	<del><b>D B</b></del>	<del><b>E B</b></del>
<b>A D</b>	<b>B D</b>	<b>C D</b>	<del><b>D C</b></del>	<del><b>E C</b></del>
<b>A E</b>	<b>B E</b>	<b>C E</b>	<b>D E</b>	<del><b>E D</b></del>

$$\begin{aligned}\text{Permutations} &= {}^5P_2 \\ &= 20\end{aligned}$$

$$\begin{aligned}\text{Combinations} &= \frac{20}{2!} \\ &= 10\end{aligned}$$

5 objects, arrange 3 of them

A B C	B <del>A</del> C	C <del>A</del> B	D <del>A</del> B	E <del>A</del> B
A B D	B <del>A</del> D	C <del>A</del> D	D <del>A</del> C	E <del>A</del> C
A B E	B <del>A</del> E	C <del>A</del> E	D <del>A</del> E	E <del>A</del> D
A <del>C</del> B	B <del>C</del> A	C <del>B</del> A	D <del>B</del> A	E <del>B</del> A
A C D	B C D	C <del>B</del> D	D <del>B</del> C	E <del>B</del> C
A C E	B C E	C <del>B</del> E	D <del>B</del> E	E <del>B</del> D
A <del>D</del> B	B <del>D</del> A	C <del>D</del> A	D <del>C</del> A	E <del>C</del> A
A <del>D</del> C	B <del>D</del> C	C <del>D</del> B	D <del>C</del> B	E <del>C</del> B
A D E	B D E	C D E	D <del>C</del> E	E <del>C</del> D
A <del>E</del> B	B <del>E</del> A	C <del>E</del> A	D <del>E</del> A	E <del>D</del> A
A <del>E</del> C	B <del>E</del> C	C <del>E</del> B	D <del>E</del> B	E <del>D</del> B
A <del>E</del> D	B <del>E</del> D	C <del>E</del> D	D <del>E</del> C	E <del>D</del> C

Permutations =  ${}^5P_3$   
 = 60

Combinations =  $\frac{60}{3!}$   
 = 10

If we have  $n$  different objects, and we arrange  $k$  of them and are not concerned about the order;

$$\begin{aligned}\text{Number of Arrangements} &= \frac{{}^n P_k}{k!} \\ &= \frac{n!}{(n-k)!k!} \\ &= {}^n C_k\end{aligned}$$

e.g. (i) How many ways can 6 numbers be chosen from 45 numbers?

$$\begin{aligned}\text{Ways} &= {}^{45} C_6 \\ &= \underline{8145060}\end{aligned}$$

*Note: at 40 cents per game, \$3 258 024 = amount of money you have to spend to guarantee a win in Lotto.*

(ii) Committees of five people are to be obtained from a group of seven men and four women.

How many committees are possible if;

a) there are no restrictions?

$$\begin{aligned}\text{Committees} &= {}^{11}C_5 \\ &= \underline{462}\end{aligned}$$

With no restrictions, choose 5 people from 11, gender does not matter

b) the committee contains only males?

$$\begin{aligned}\text{Committees} &= {}^7C_5 \\ &= \underline{21}\end{aligned}$$

By restricting it to only males, there is only 7 people to choose from

c) the committee contains at least one woman?

$$\begin{aligned}\text{Committees} &= 462 - 21 \\ &= \underline{441}\end{aligned}$$

easier to work out male only and subtract from total number of committees

(iii) A hand of five cards is dealt from a regular pack of fifty two cards.

a) What is the number of possible hands?

$$\begin{aligned}\text{Hands} &= {}^{52}C_5 \\ &= \underline{2598960}\end{aligned}$$

b) What is the probability of getting “three of a kind”?

choose which number has "three of a kind"      choose three of those cards

Hands =  ${}^{13}C_1 \times {}^4C_3 \times {}^{48}C_2$       choose remaining two cards from the rest

$$= \underline{58656}$$

$$\begin{aligned}P(\text{three of a kind}) &= \frac{58656}{2598960} \\ &= \underline{\frac{94}{2915}} \quad (=3.2\%) \end{aligned}$$

## 2004 Extension 1 HSC Q2e)

A four person team is to be chosen at random from nine women and seven men.

(i) In how many ways can this team be chosen?

$$\begin{aligned}\text{Teams} &= {}^{16}C_4 \\ &= \underline{1820}\end{aligned}$$

With no restrictions, choose 4 people from 16, gender does not matter

(ii) What is the probability that the team will consist of four women?

$$\begin{aligned}\text{Teams} &= {}^9C_4 \\ &= 126\end{aligned}$$

By restricting it to only women, there is only 9 people to choose from

$$\begin{aligned}P(4 \text{ women team}) &= \frac{126}{1820} \\ &= \underline{\frac{9}{130}}\end{aligned}$$

**Exercise 10G; odd**  
*(not 19, 27)*