

(6) t results

$$\text{If } t = \tan \frac{x}{2}$$

$$\text{then } \sin x = \frac{2t}{1+t^2}, \quad \cos x = \frac{1-t^2}{1+t^2}$$

$$\begin{aligned} \frac{dt}{dx} &= \frac{1}{2} \sec^2 \frac{x}{2} \\ &= \frac{1}{2} \left(1 + \tan^2 \frac{x}{2} \right) \\ &= \frac{1}{2} (1 + t^2) \end{aligned}$$

$$\frac{dx}{dt} = \frac{2}{1+t^2}$$

$$dx = \frac{2dt}{1+t^2}$$

$$\text{e.g. } \int \frac{dx}{7 + 6 \cos x}$$

$$= \int \frac{2dt}{7 + 6 \left(\frac{1-t^2}{1+t^2} \right)}$$

$$= \int \frac{2dt}{7 + 7t^2 + 6 - 6t^2}$$

$$= \int \frac{2dt}{13 + t^2}$$

$$= \frac{2}{\sqrt{13}} \tan^{-1} \frac{t}{\sqrt{13}} + c$$

$$= \frac{2}{\sqrt{13}} \tan^{-1} \left(\frac{\tan \frac{x}{2}}{\sqrt{13}} \right) + c$$

$$t = \tan \frac{x}{2}$$

$$dx = \frac{2dt}{1+t^2}$$

If $t = \tan x$

$$\frac{dt}{dx} = \sec^2 x$$

$$= 1 + \tan^2 x$$

$$= 1 + t^2$$

$$\frac{dx}{dt} = \frac{1}{1+t^2}$$

$$dx = \frac{dt}{1+t^2}$$

In General :

If $t = \tan \frac{x}{a}$

$$\frac{dt}{dx} = \frac{1}{a} \sec^2 \frac{x}{a}$$

$$= \frac{1}{a} \left(1 + \tan^2 \frac{x}{a} \right)$$

$$= \frac{1}{a} (1 + t^2)$$

$$\frac{dx}{dt} = \frac{a}{1+t^2}$$

$$dx = \frac{adt}{1+t^2}$$

Exercise 2C; 20 21, 24, 25